Exact solutions for quenches in the Lieb-Liniger Bose gas and Heisenberg spin chains

RAQIS conference
Dijon, 4 September 2014

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Universiteit van Amsterdam

Work done in collaboration with (among others):

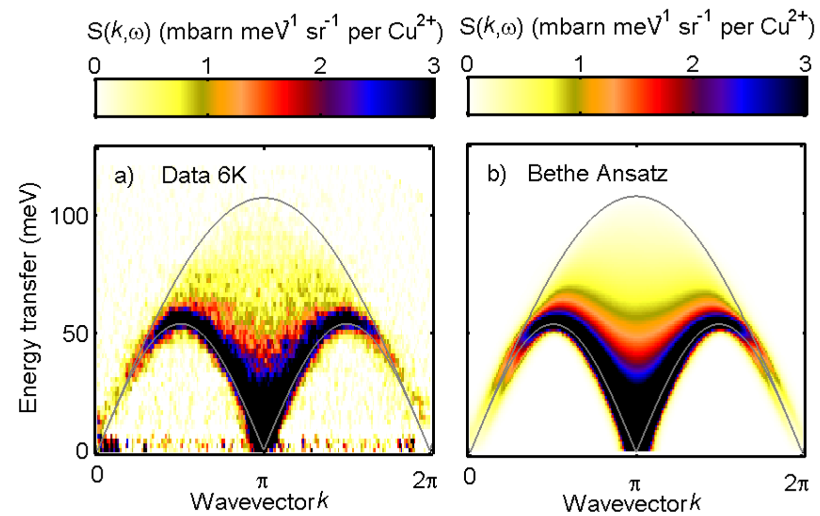
A'dam gang: M. Panfil, S. Eliens, T. Fokkema, J. De Nardis, B. Wouters, R. van den Berg, R. Vlijm, G. Brandino, M. Brockmann, D. Fioretto, O. El Araby, F.H.L. Essler, V. Gritsev, R. Konik

Plan of the talk

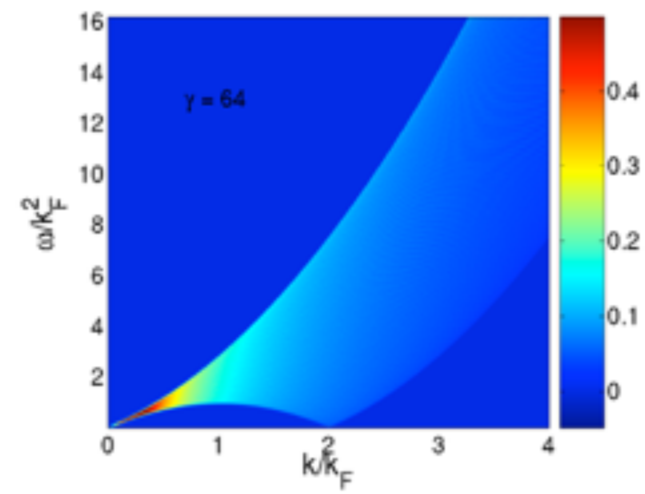
- Equilibrium dynamics
  - *Finite temperatures in Lieb-Liniger*
  - *Spinons in Heisenberg*
- Out-of-equilibrium dynamics
  - *Quantum dots*
  - *Interaction quench in Lieb-Liniger*
  - *Anisotropy quench in XXZ*
- Summary & perspectives

The quench action

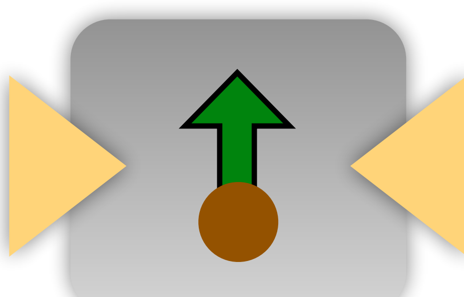
Applications of integrability in many-body physics



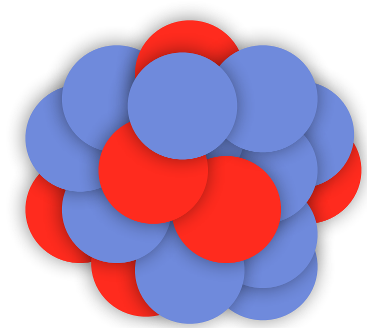
Quantum magnetism



Ultracold atoms

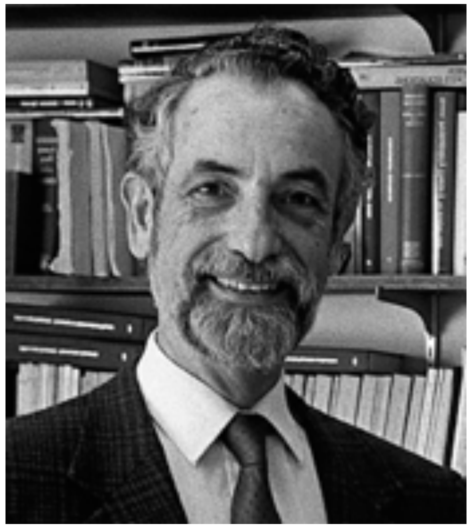


Quantum dots,
NV centers



Atomic nuclei

Models discussed in this talk:



- Interacting Bose gas (Lieb-Liniger)

$$\mathcal{H}_N = -\sum_{j=1}^{N} \frac{\partial^2}{\partial x_j^2} + 2c \sum_{1\le j<l\le N} \delta(x_j - x_l)$$

- Heisenberg spin-1/2 chain

$$H = \sum_{j=1}^{N} \left[J(S_j^x S_{j+1}^x + S_j^y S_{j+1}^y + \Delta S_j^z S_{j+1}^z) - H_z S_j^z \right]$$

- Central spin (Gaudin magnet)

$$H_{\rm int} = B_z S_0^z + \sum_{j=1}^{N} A_j \, \vec{S}_0 \cdot \vec{I}_j$$

The
Bethe Wavefunction

Michel Gaudin's book *La fonction d'onde de Bethe* is a uniquely influential masterpiece on exactly solvable models of quantum mechanics and statistical physics. Available in English for the first time, this translation brings his classic work to a new generation of graduate students and researchers in physics. It presents a mixture of mathematics interspersed with powerful physical intuition, retaining the author's unmistakably honest tone.

The book begins with the Heisenberg spin chain, starting from the coordinate Bethe Ansatz and culminating in a discussion of its thermodynamic properties. Delta-interacting bosons (the Lieb-Liniger model) are then explored, and extended to exactly solvable models associated with a reflection group. After discussing the continuum limit of spin chains, the book covers six- and eight-vertex models in extensive detail, from their lattice definition to their thermodynamics. Later chapters examine advanced topics such as multicomponent delta-interacting systems, Gaudin magnets and the Toda chain.

MICHEL GAUDIN is recognized as one of the foremost experts in this field, and has worked at Commissariat à l'énergie atomique (CEA) and the Service de Physique Théorique, Saclay. His numerous scientific contributions to the theory of exactly solvable models are well known, including his famous formula for the norm of Bethe wavefunctions.

JEAN-SÉBASTIEN CAUX is a Professor in the theory of low-dimensional quantum condensed matter at the University of Amsterdam. He has made significant contributions to the calculation of experimentally observable dynamical properties of these systems.

Cover illustration: a representation of the Yang-Baxter relation by John Collingwood.

Cover designed by Hart McLeod Ltd

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Gaudin and Caux The Bethe Wavefunction

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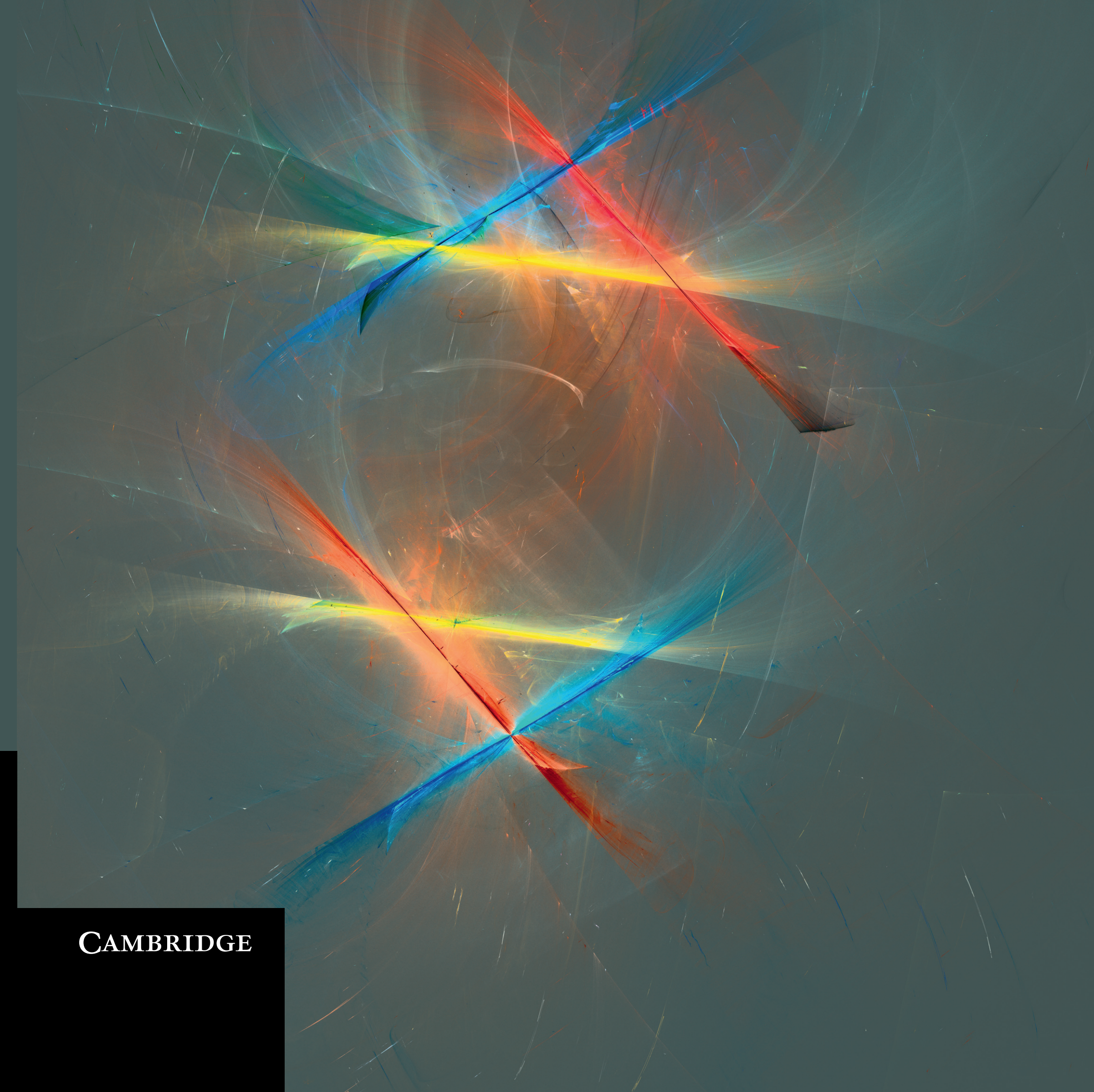
The
Bethe Wavefunction

Michel Gaudin

Translated by Jean-Sébastien Caux



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The general idea, simply stated:

Start with your favourite quantum state
(expressed in terms of Bethe states)

$$\mathcal{O} \Rightarrow |\{\lambda\}\rangle$$

Apply some operator on it

Reexpress the result in the basis of Bethe states:

$$\mathcal{O}|\{\lambda\}\rangle = \sum_{\{\mu\}} F^{\mathcal{O}}_{\{\mu\},\{\lambda\}} |\{\mu\}\rangle$$

using 'matrix elements' $F^{\mathcal{O}}_{\{\mu\},\{\lambda\}} = \langle\{\mu\}|\mathcal{O}|\{\lambda\}\rangle$

Equilibrium dynamics from integrability

Equilibrium dynamics using integrability

Cases which we've handled:

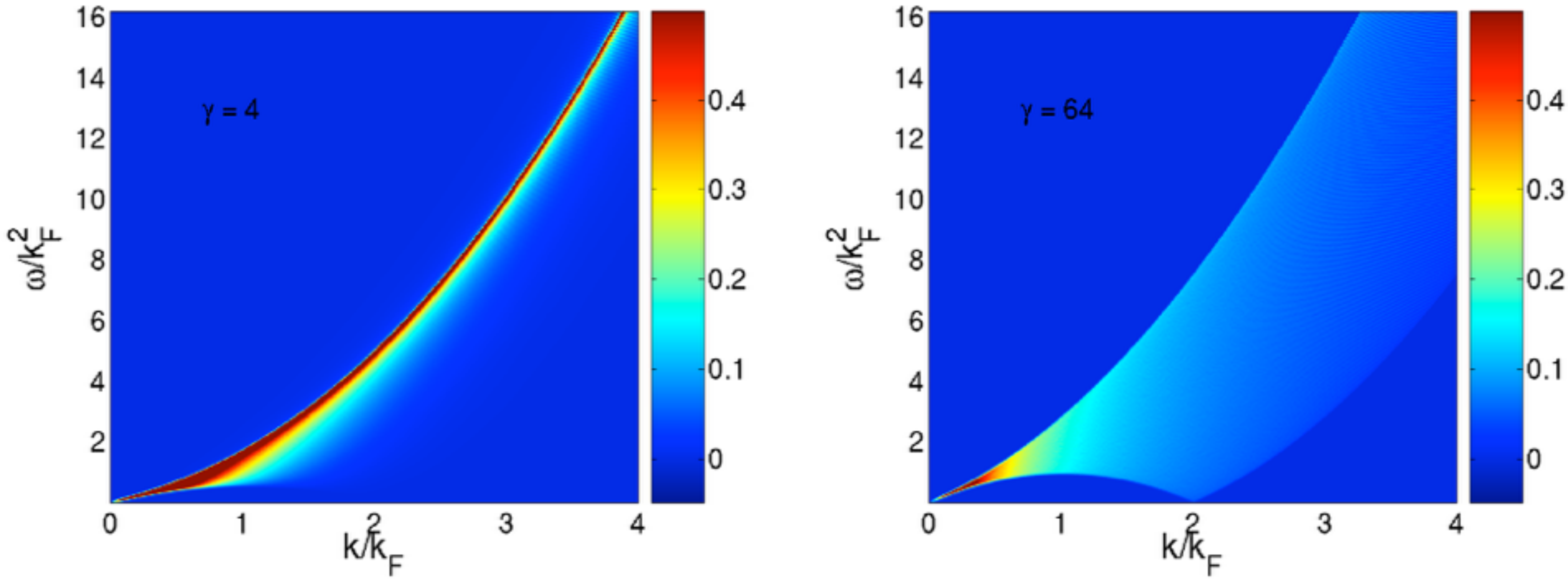
- Heisenberg model
- XXZ gapless antiferromagnets
- XXZ gapped antiferromagnets
- Spin-1 chain (Babujan-Takhtajan)
- Lieb-Liniger (repulsive interactions)
- Lieb-Liniger (attractive interactions)
- Richardson model
- Central spin model
- Perturbed integrable models: NRG from BA

Repulsive Lieb-Liniger gas

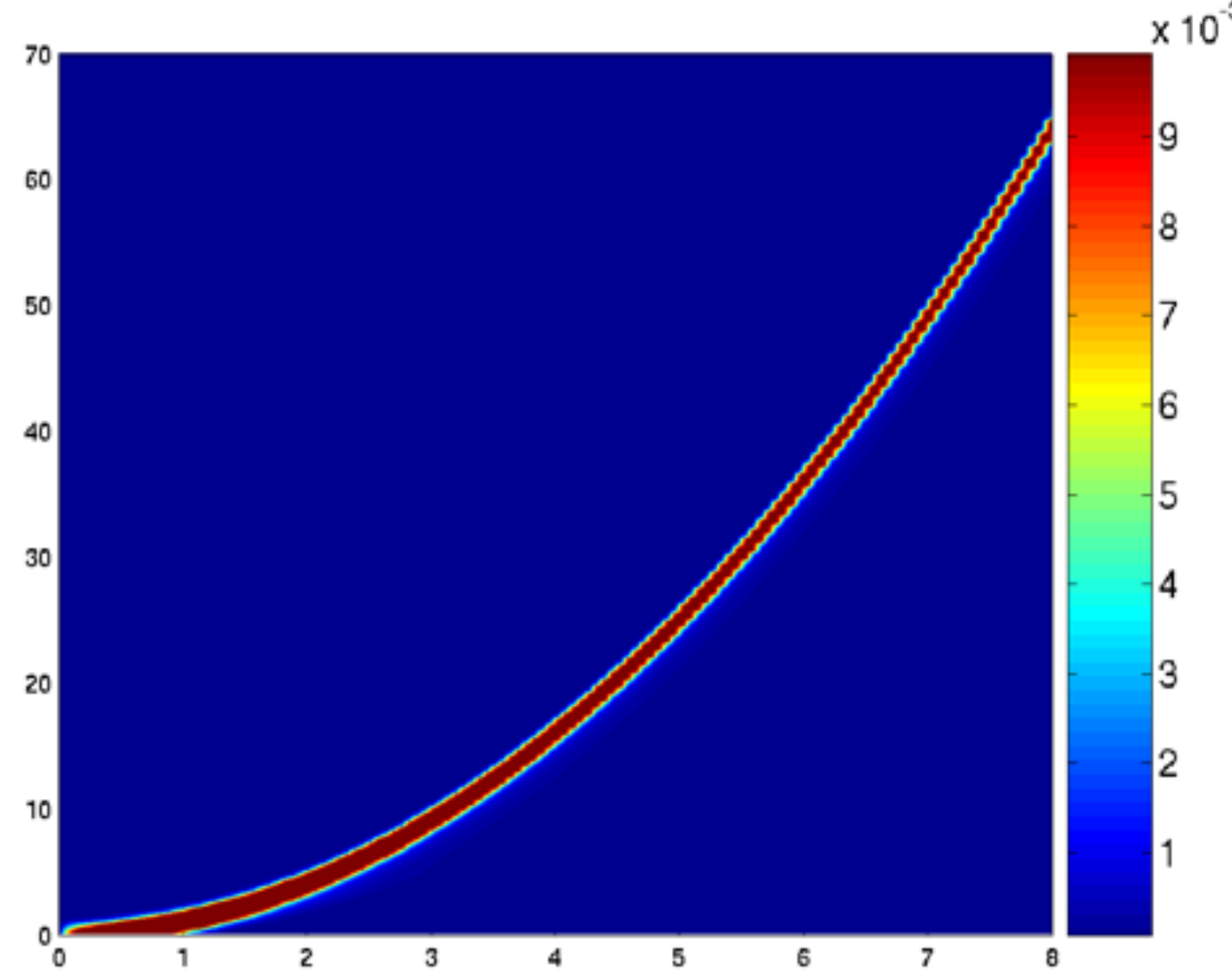
Density-density (dynamical SF)

(J-S C & P Calabrese, PRA 2006)

$$S(k,\omega) = \frac{2\pi}{L}\sum_{\alpha} |\langle 0|\rho_k|\alpha\rangle|^2 \delta(\omega - E_\alpha + E_0)$$

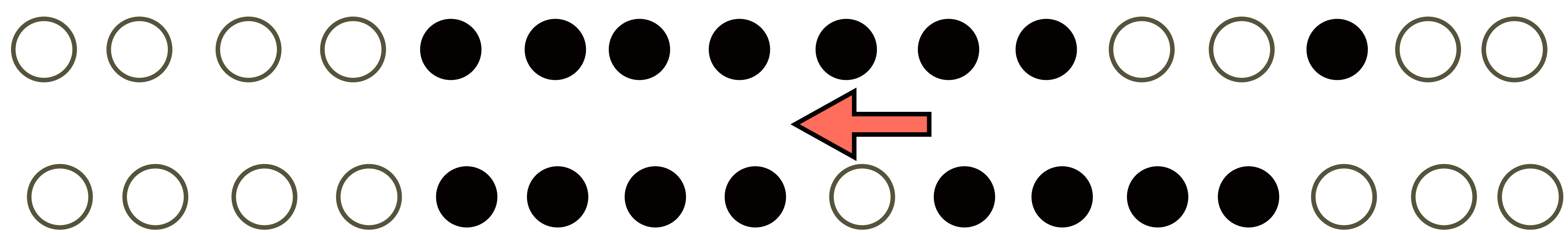


Correspondence with excitations

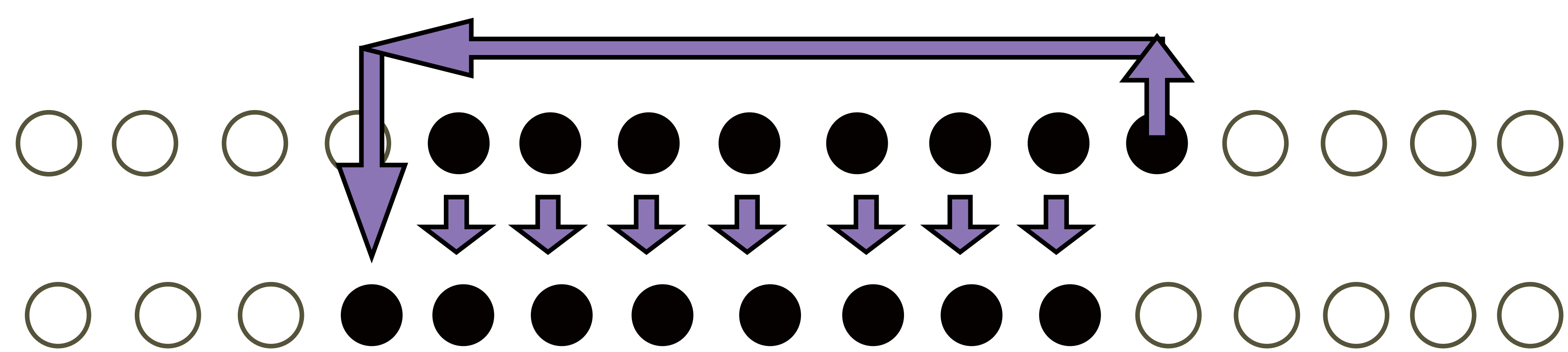


Particle-like

Hole-like



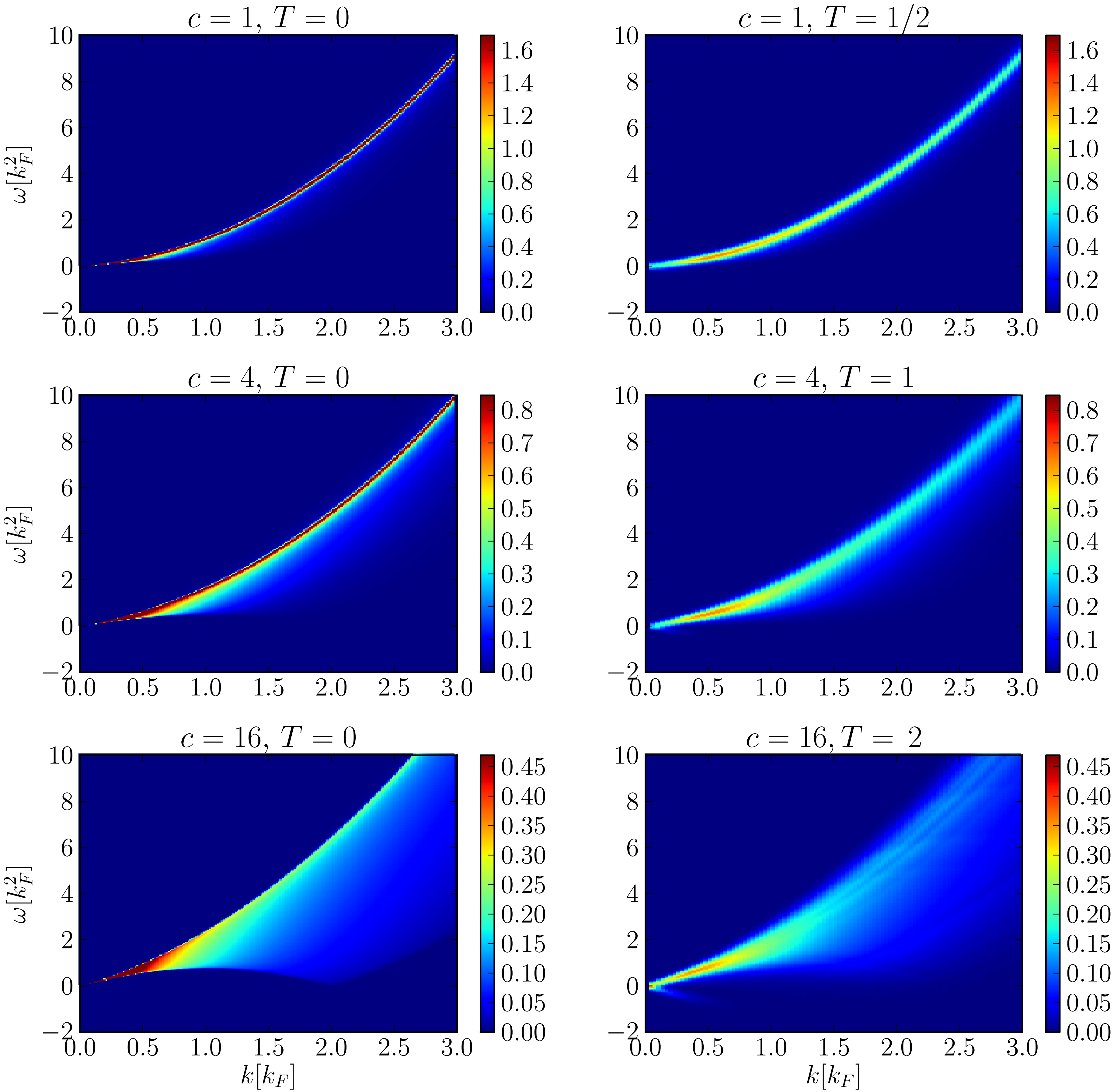
Umklapp



Repulsive Lieb-Liniger gas

Dynamical structure factor at finite T

M. Panfil and J-SC, PRA 89 (2014)

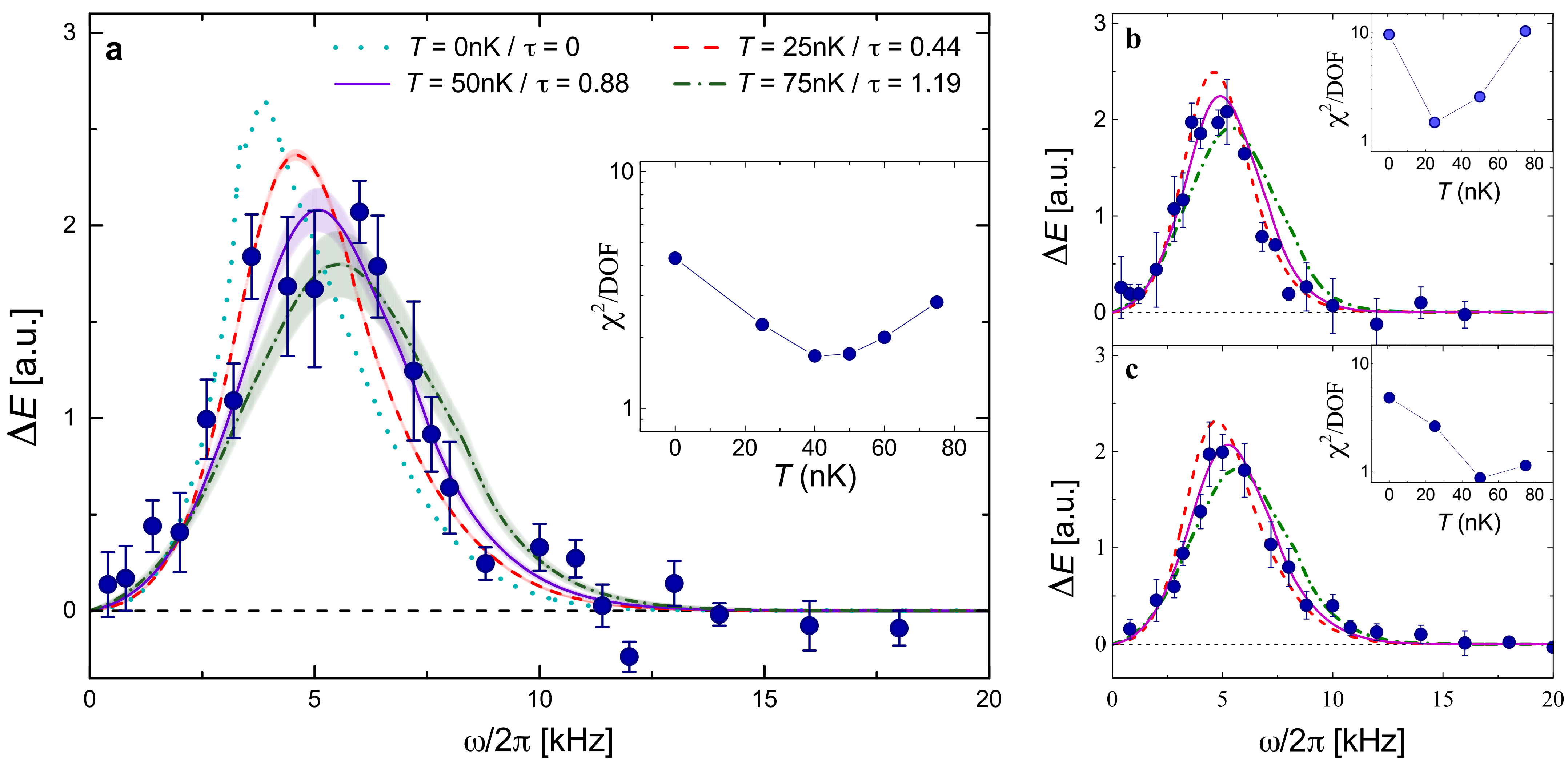


Cold atoms

Observing elementary excitations of correlated one-dimensional Bose gases

N. Fabbri, M. Panfil, D. Clément, L. Fallani, M. Inguscio, C. Fort and J.-S. Caux
arxiv:1406.2176

Density correlations using Bragg spectroscopy



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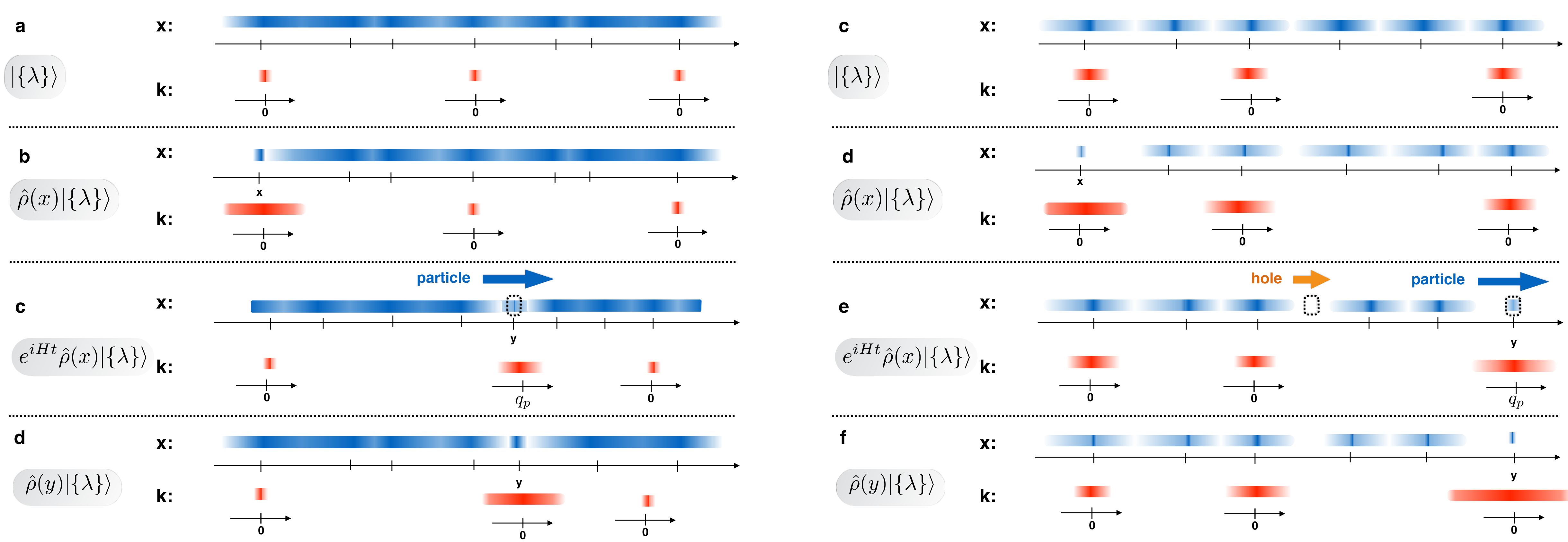
arxiv:1406.2176

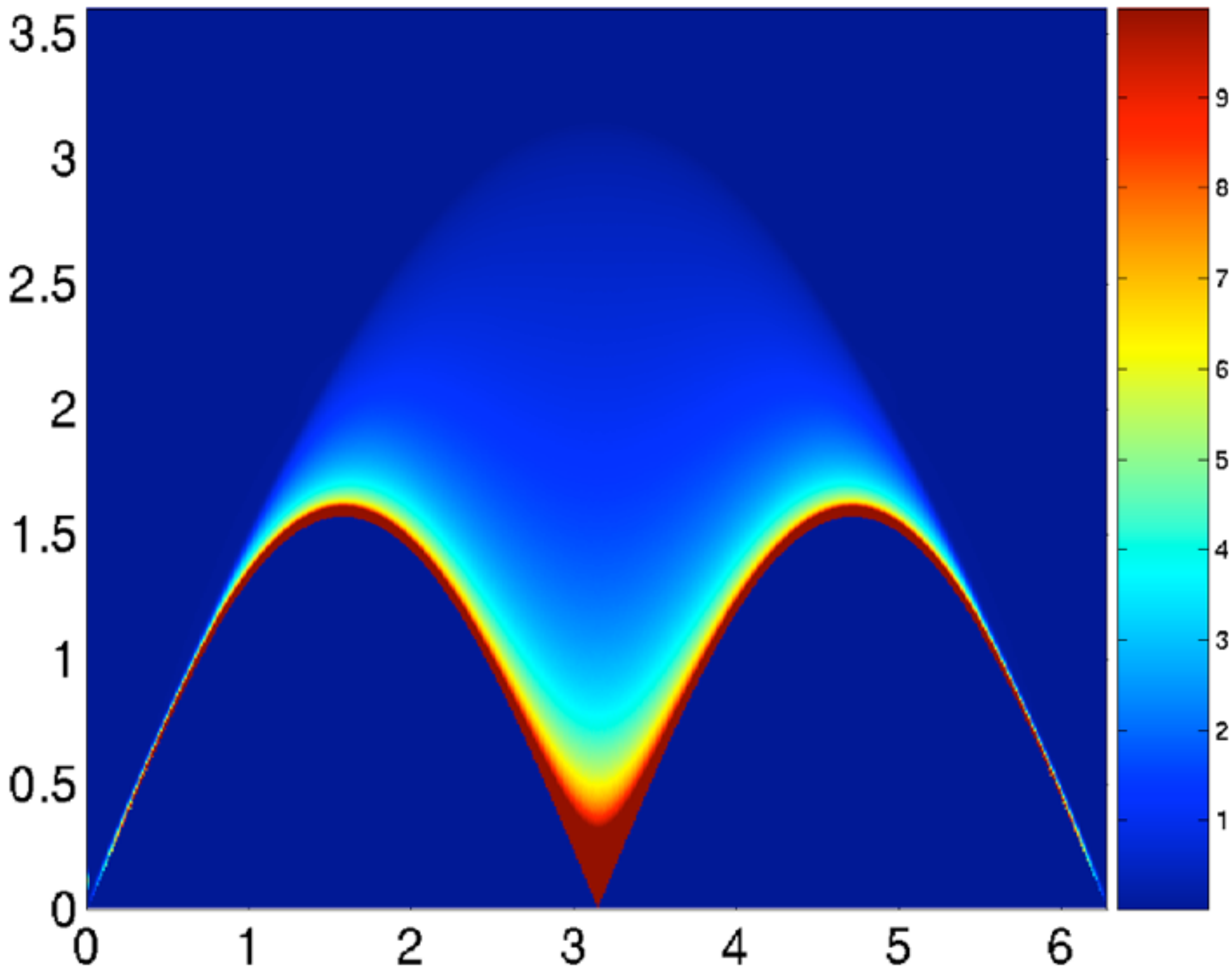
Intuitive picture of correlations: from ‘quasiparticles’

weak interactions | strong interactions

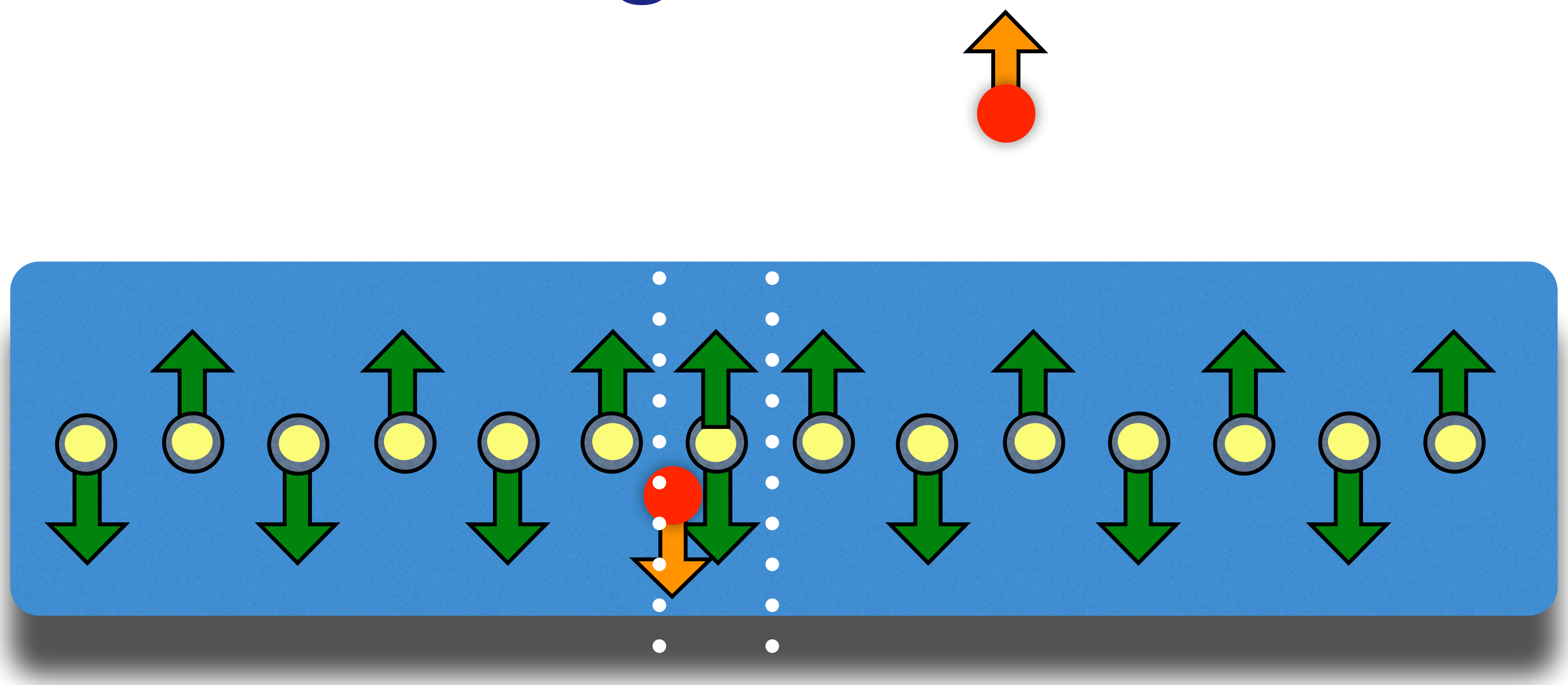


Heisenberg spin chain

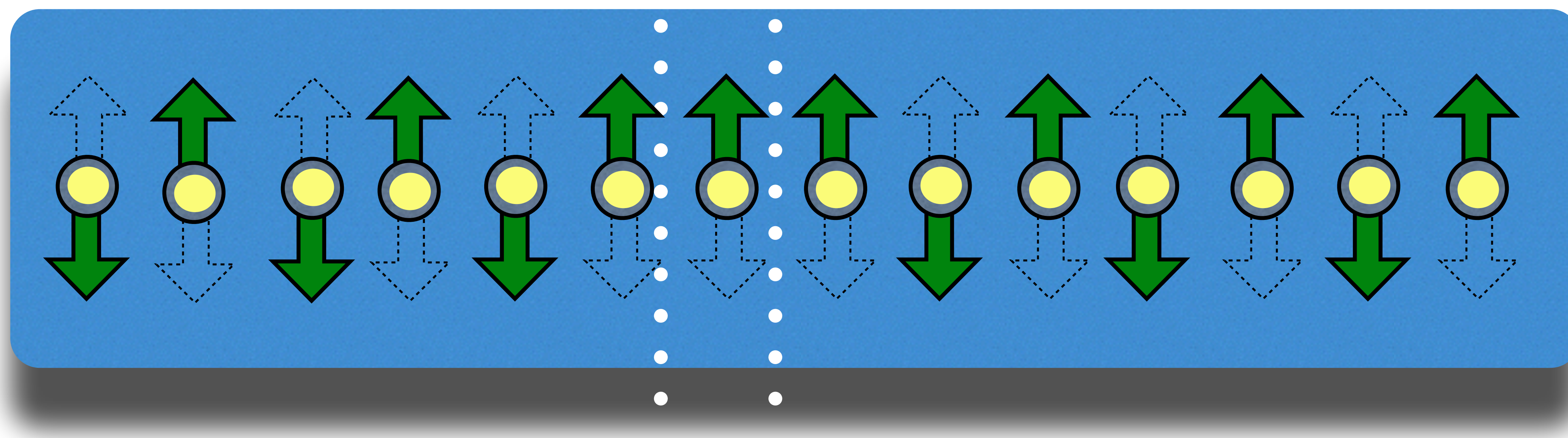
$$S(k,\omega), \quad \Delta = 1, \quad h = 0$$



Investigating spin chains using neutrons



Investigating spin chains using neutrons

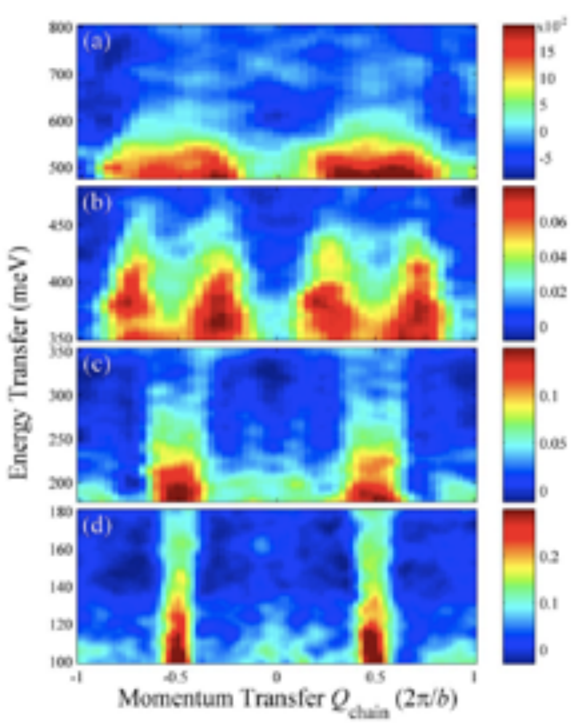


Effective particles: ***spinons***

Quantum spin chains

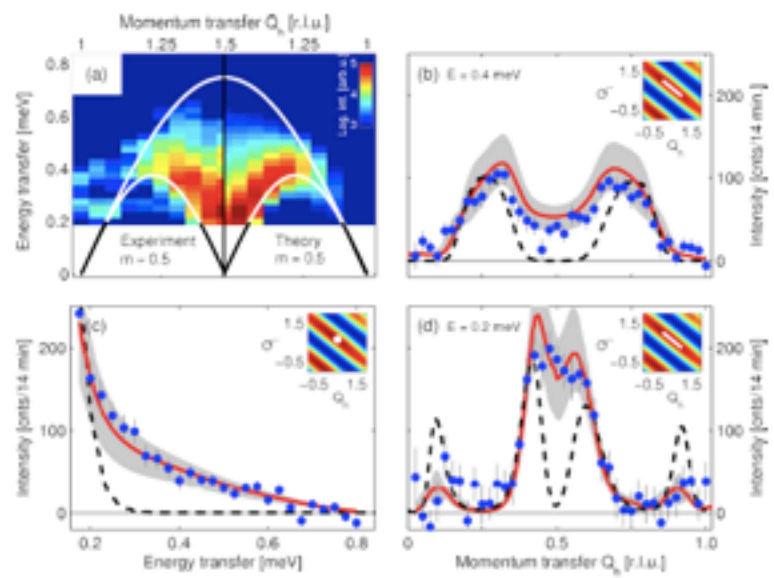
Correlations, experiments (INS, RIXS), prefactors, ...

Sr_2CuO_3



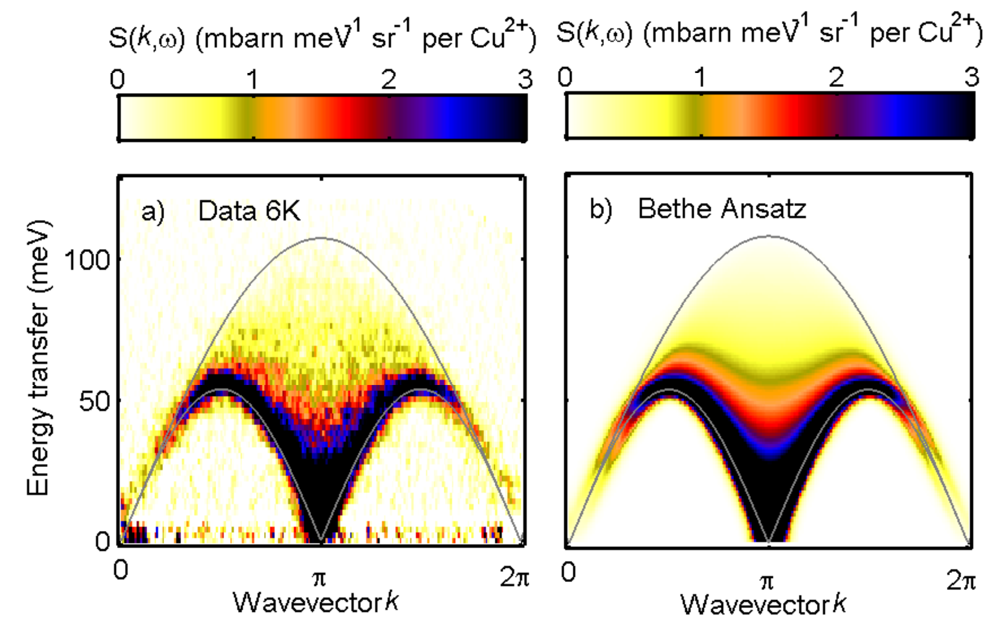
Walters, Perring, Caux, Savici, Gu, Lee, Ku, Zaliznyak, NATURE PHYSICS 2009

$(C_5D_{12}N)_2CuBr_4$

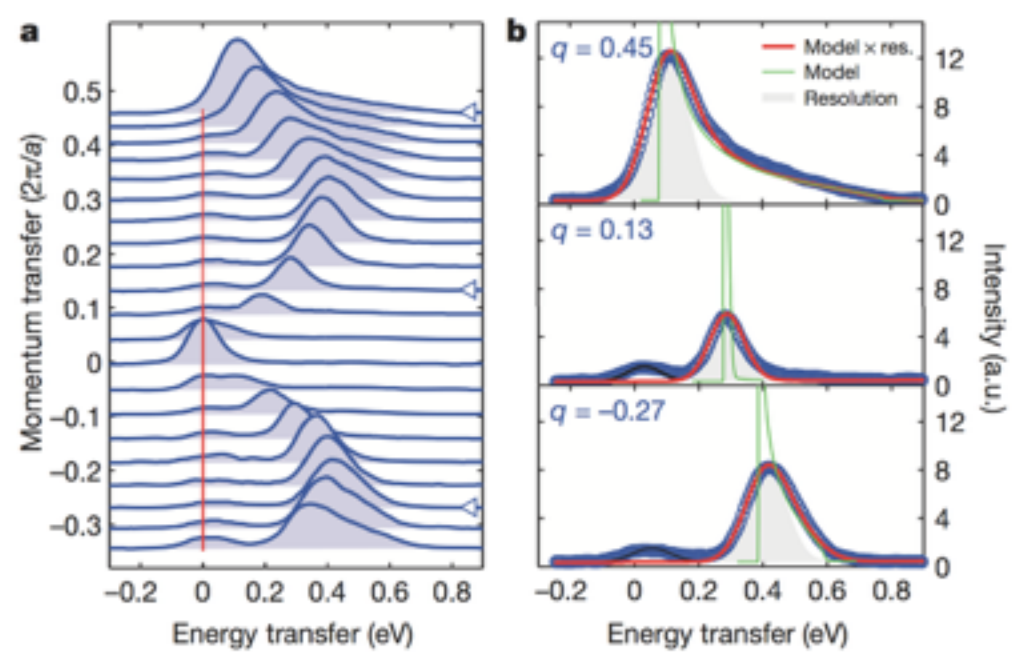


Thielemann, Rüegg, Rønnow, Läuchli, Caux, Normand, Biner, Krämer, Güdel, Stahn, Habicht, Kiefer, Boehm, McMorrow, Mesot, PRL 2009

$KCuF_3$



Lake, Tennant, Caux, Barthel, Schollwöck, Nagler, Frost, PRL 2013



Sr_2CuO_3

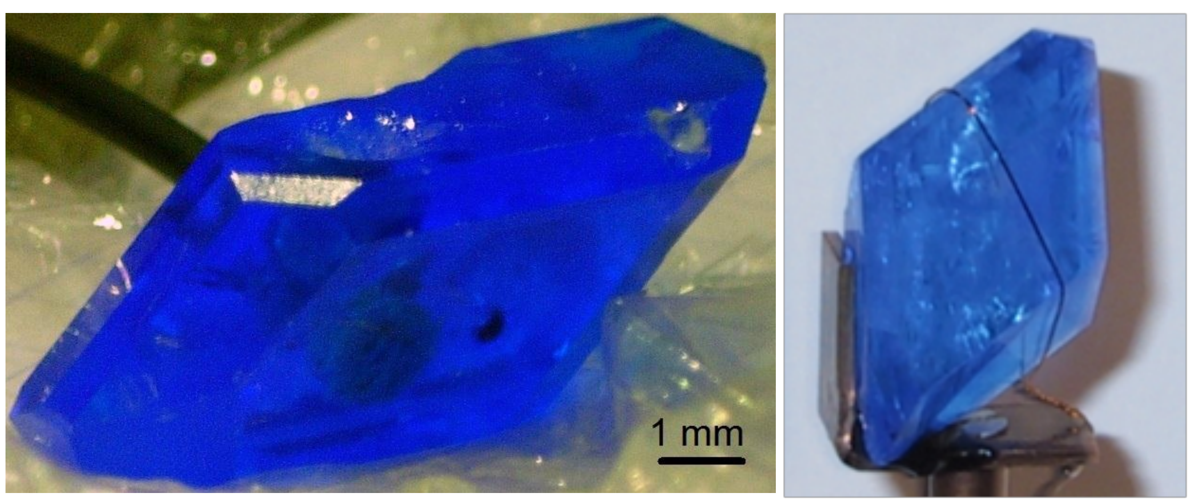
(RIXS)

Schlappa, Wohlfeld, Zho, Mourigal, Haverkort, Strocov, Hozoi, Monney, Nishimoto, Singh, Revcolevschi, Caux, Patthey Rønnow, van den Brink, Schmitt, NATURE 2012

Counting fractional spinon excitations in the quantum Heisenberg antiferromagnetic chain

Martin Mourigal,[1,2,3,*] Mechthild Enderle,[1] Axel Klöpperpieper,[4]
Jean-Sébastien Caux,[5] Anne Stunault,[1] and Henrik M. Rønnow[2]

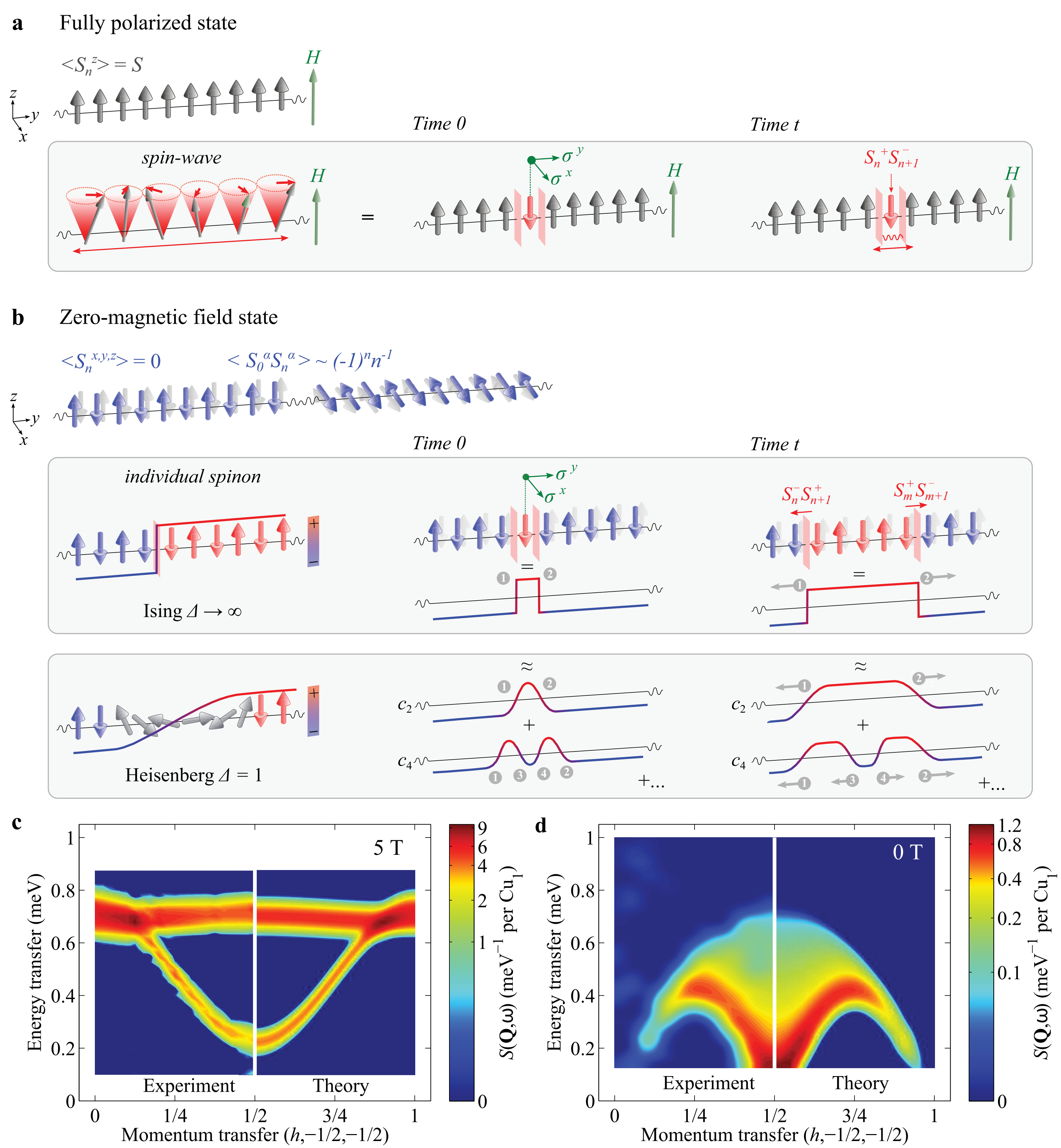
(Nature Physics 2013)



$\mathrm{CuSO_4 \cdot 5D_2O}$

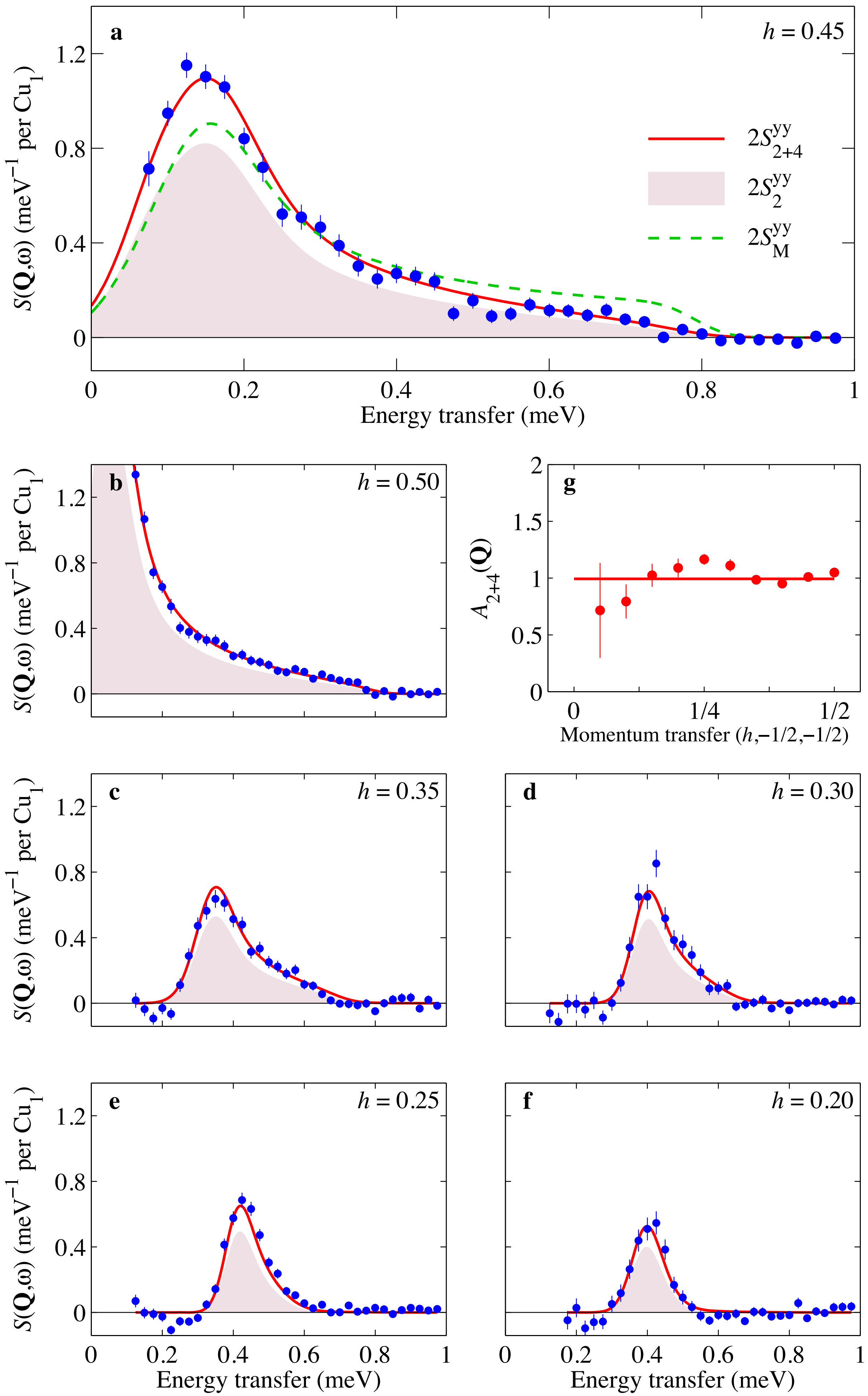
Roger Hiorns, *Seizure*

Multi-spinon processes: intuitive picture

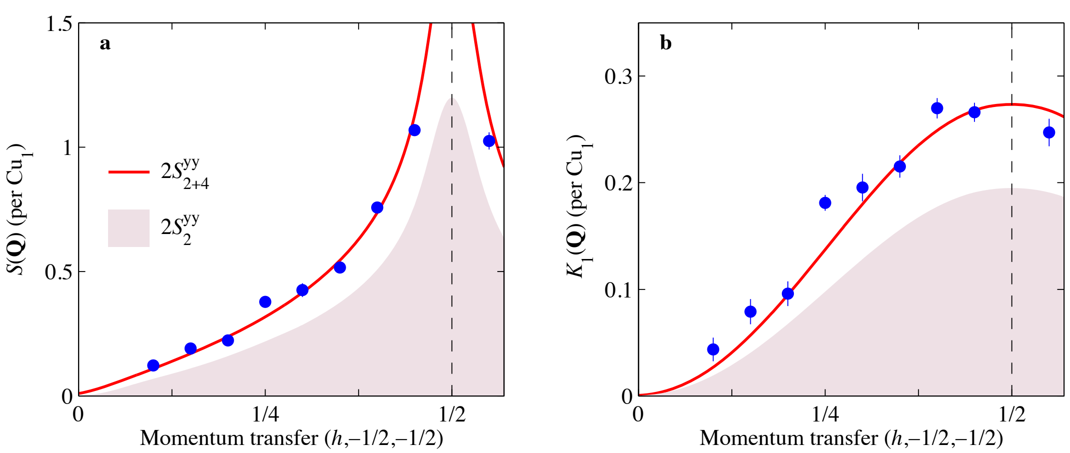


Experimental data is accurate enough to discriminate between:

- Müller ansatz (off)
- BA: 2 spinons (off)
- BA: 2 + 4 spinons (on!)

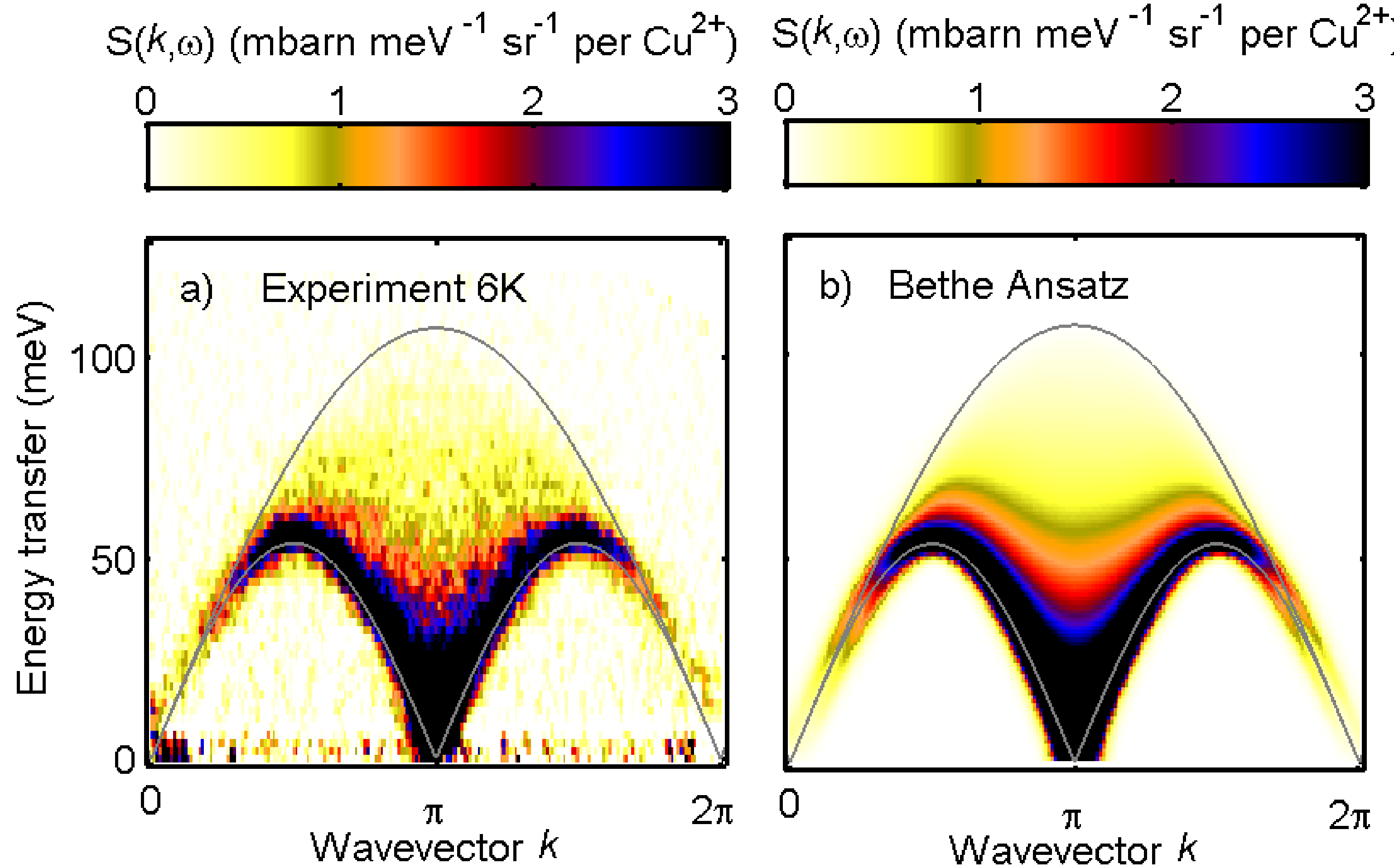


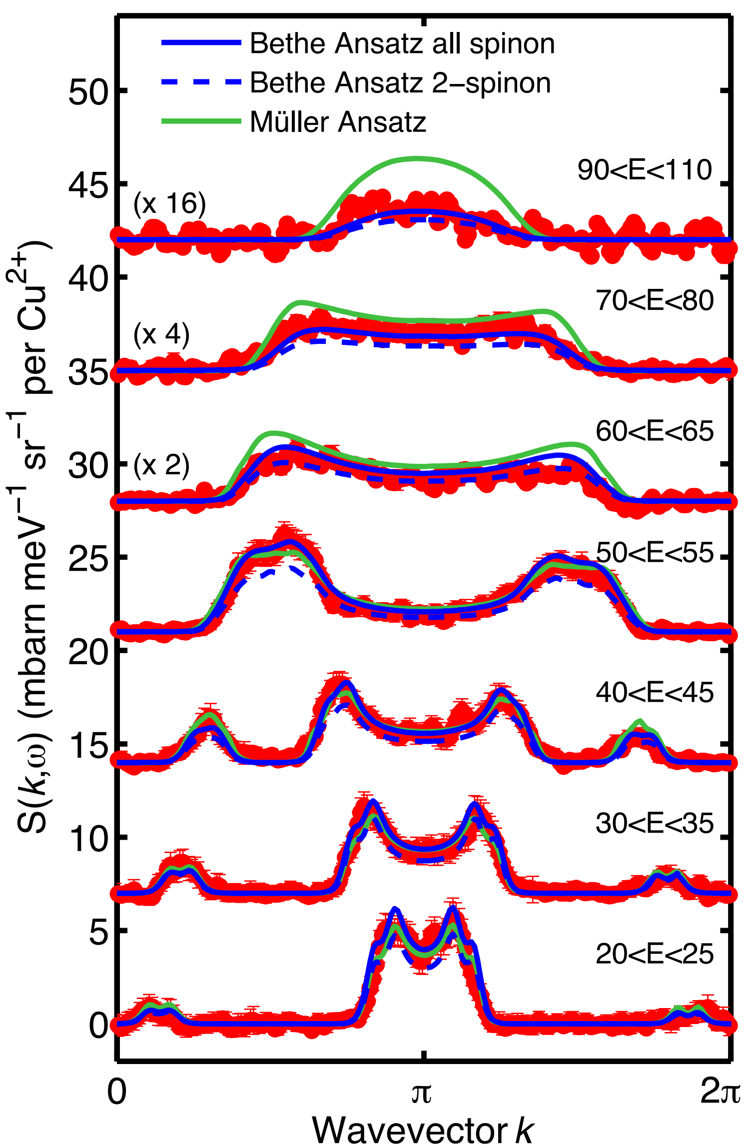
Frequency-integrated:

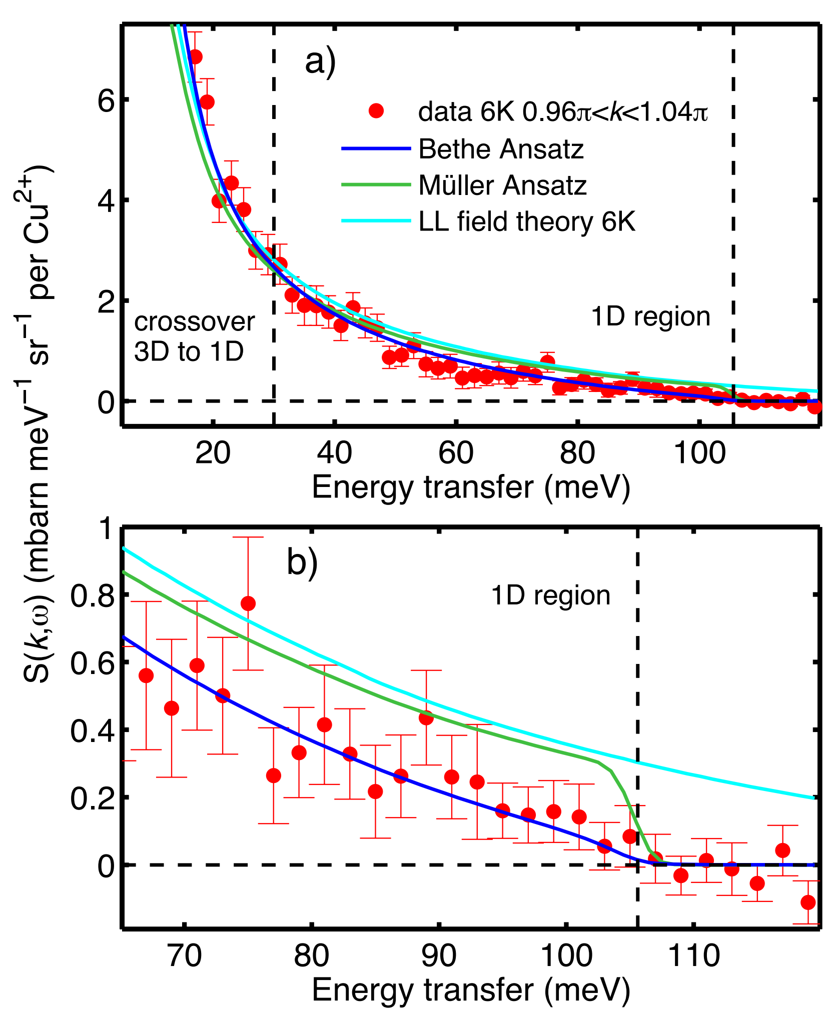


$KCuF_3$

Lake, Tennant, Caux, Barthel, Schollwöck, Nagler, Frost PRL 2013







Experiment accurate enough to discriminate between:

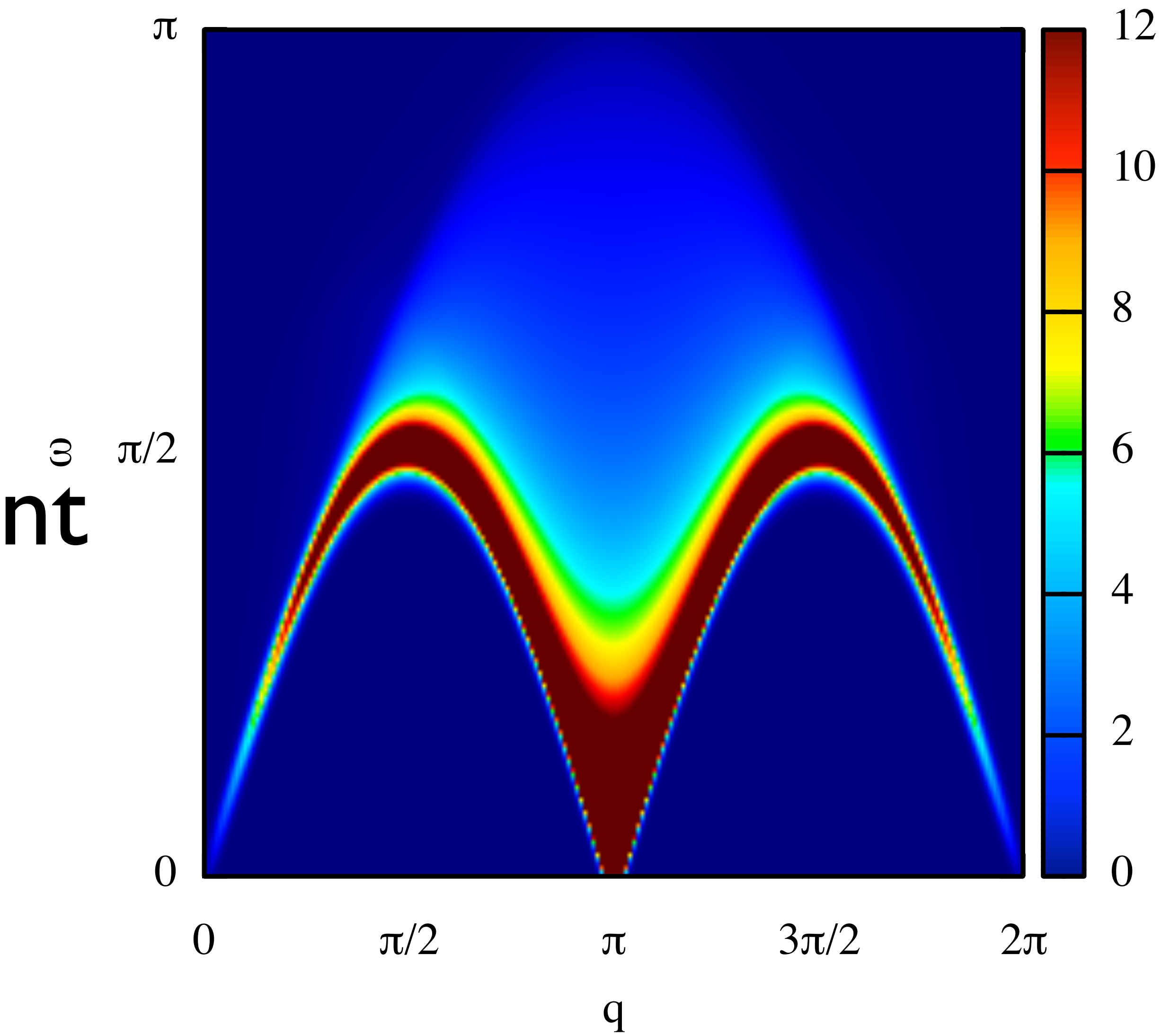
- Müller Ansatz
- Field theory
- Bethe Ansatz (best)

Babujan-Takhtajan spin-1 chain

$$H = J \sum_{j=1}^{N} \left[\mathbf{S}_j \cdot \mathbf{S}_{j+1} - (\mathbf{S}_j \cdot \mathbf{S}_{j+1})^2 - H_z S_j^z \right]$$

R. P. Vlijm and J.-S. C., JSTAT 2014

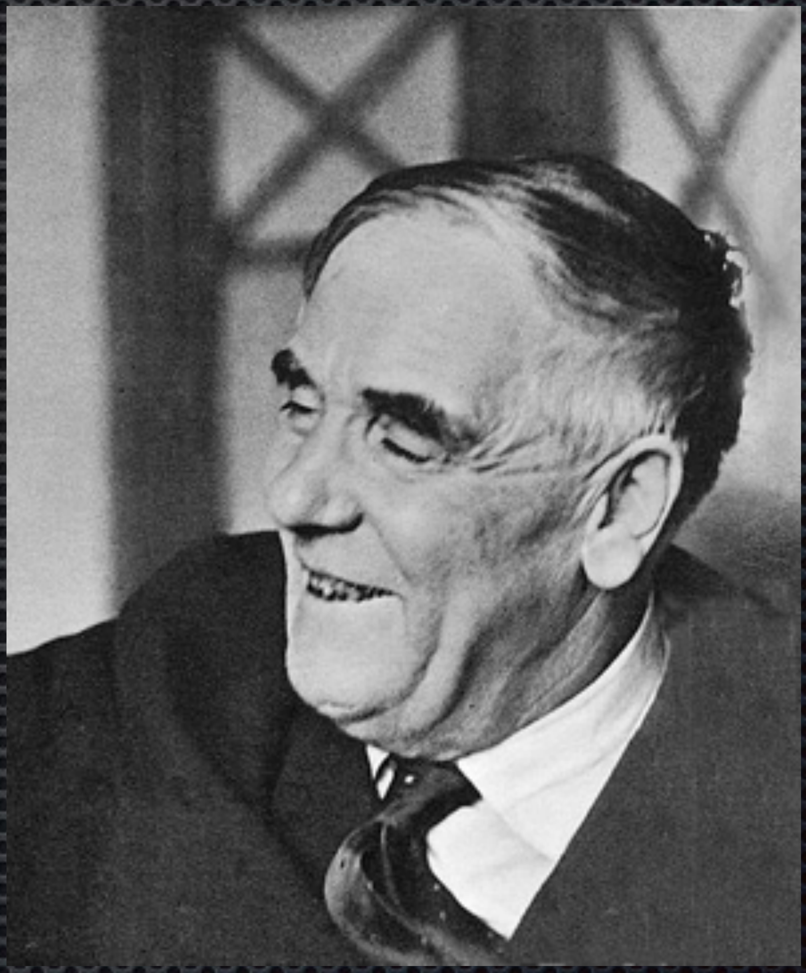
- Ground state: correlated gas of 2-strings
- String deformations: must be taken into account
- Correlations: skewed to higher energies (as compared to S=1/2)



Out-of-equilibrium dynamics from integrability

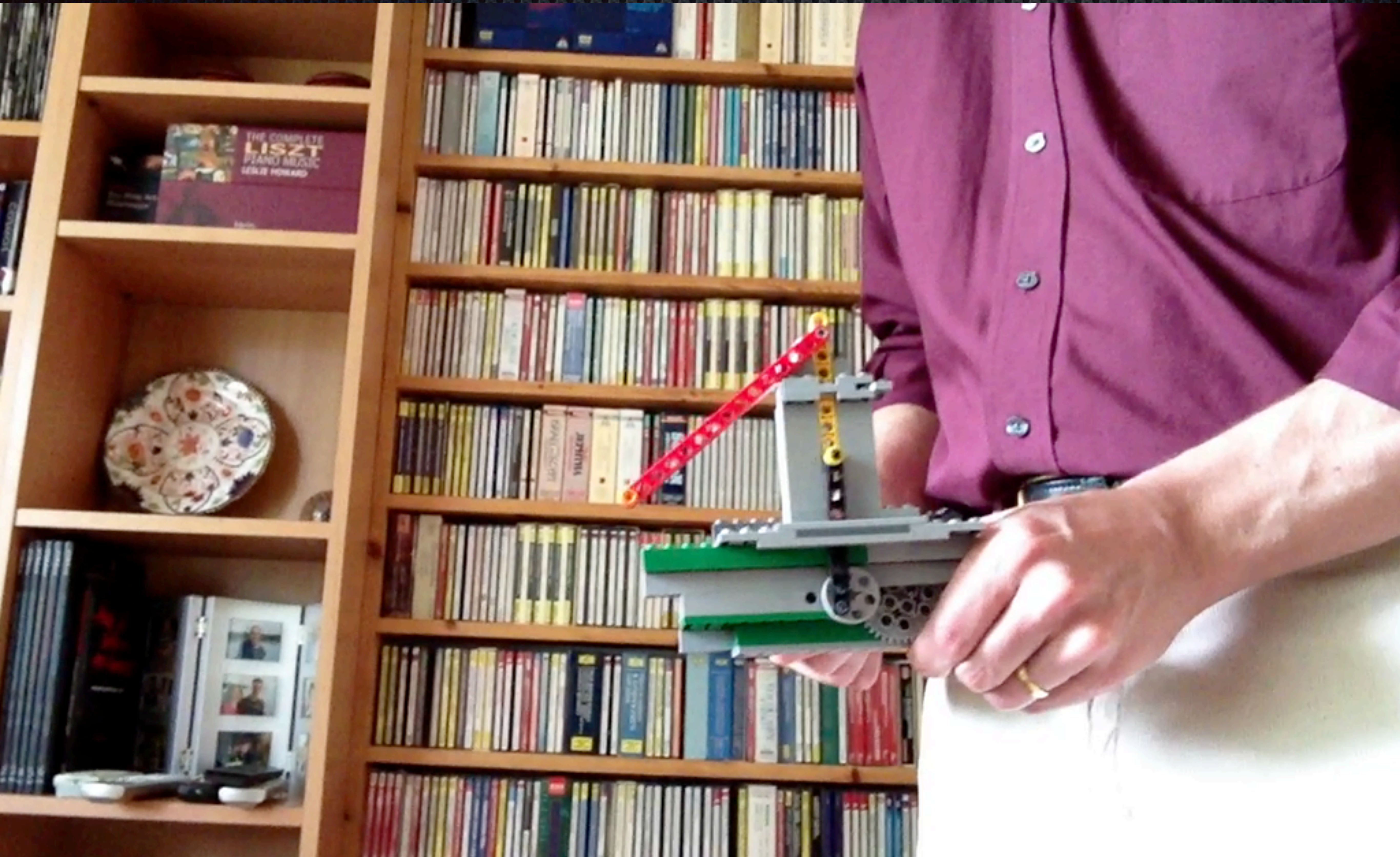
The simple pendulum on its head

Kapitsa pendulum, 1951



Pyotr L. Kapitsa
(8/7/1894-8/4/1984)

The Kapitza pendulum



Out-of-equilibrium using integrability

It's possible to treat some situations using BA

Highly excited initial (eigen)states:

- The super Tonks-Girardeau gas
- Split Fermi sea in Lieb-Liniger

Quenched states:

- Interaction quench in Richardson
- Domain wall release in Heisenberg
- Geometric quench
- Interaction turnoff in Lieb-Liniger
- Release of trapped Lieb-Liniger
- BEC to Lieb-Liniger quench
- Néel to XXZ quench

Driven systems:

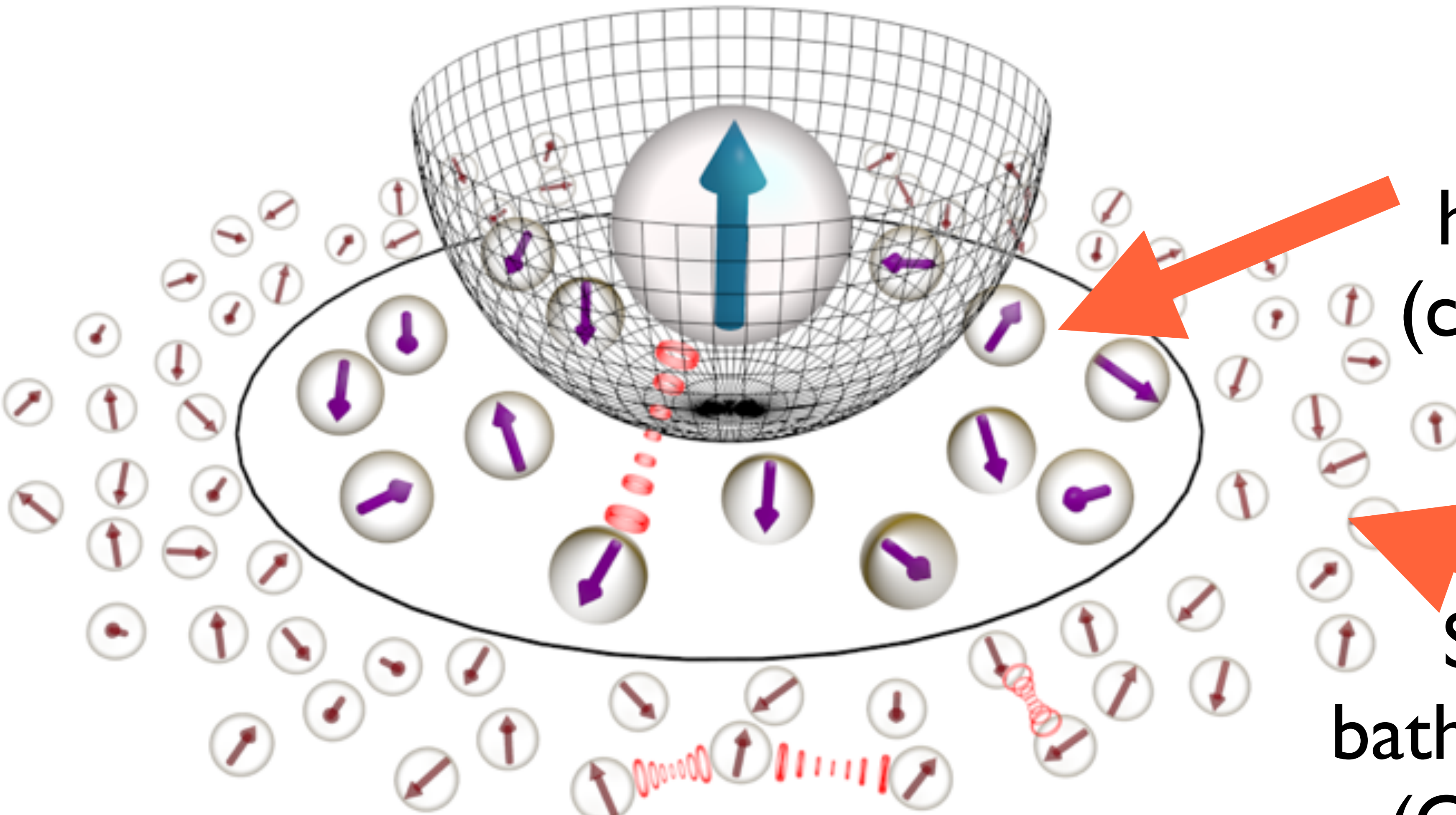
- Spin echo in quantum dots

Driven systems:

Dynamics in quantum dots

Central spin model for quantum dot

R. van den Berg et al., arxiv:1407.6503



Nearby spins: full hyperfine interaction (couplings ~ el. wavefn)

Spins further away: bath, mean-field dynamics (Ornstein-Uhlenbeck)

$$H_{\text{int}} = B_z S_0^z + \sum_{j=1}^{N} A_j \, \vec{S}_0 \cdot \vec{I}_j$$

$$V(t) = B(t) S_0^z$$

$$A_k = \frac{A}{n_0} |\psi(r_k)|^2 = \frac{A}{N} \exp\left[-\frac{(k-1)}{N_0} \right]$$

$$\langle B(t) B(0) \rangle = b^2 \exp(-Rt)$$

Usual spin echo sequence $\pi_y/2 - \tau - \pi_x - \tau - \pi_y/2$

here focus on $\tau - \pi_x - \tau$ and assume ideal pulses

Time evolution: 2nd-order Suzuki-Trotter decomposition

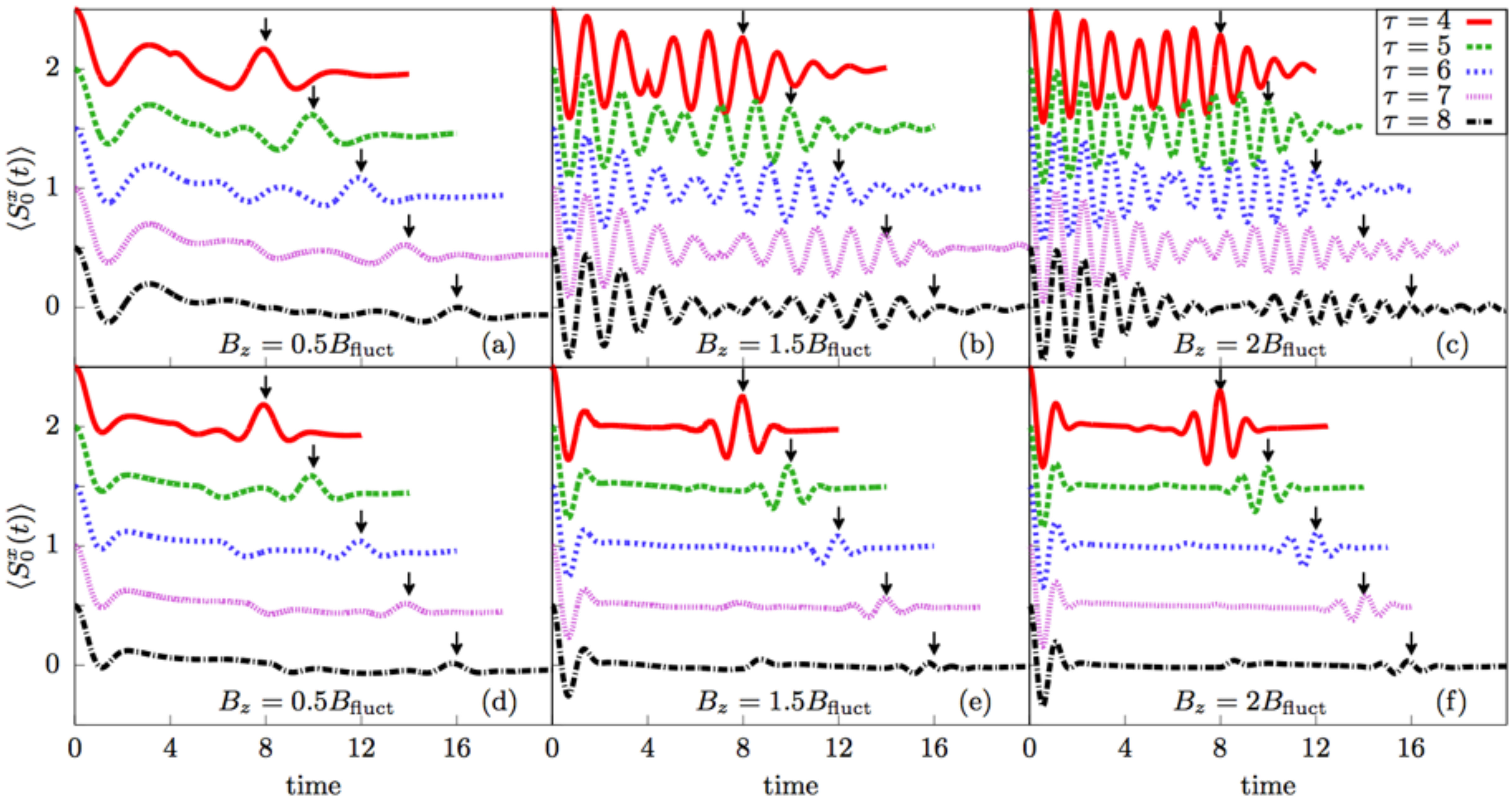
$$U(t, t+dt) \approx U^{(2)} = e^{-i\frac{dt}{2}H_{\text{int}}} e^{-idtV\left(t+\frac{dt}{2}\right)} e^{-i\frac{dt}{2}H_{\text{int}}}$$

Simulate two types of initial states:

Néel bath:
$$|\psi_0\rangle = \frac{1}{\sqrt{2}}(|\Uparrow\rangle + |\Downarrow\rangle) \otimes |\uparrow\downarrow\uparrow\downarrow\uparrow\downarrow\ldots\rangle$$

Random bath:
$$|\psi_0\rangle = \frac{1}{\sqrt{2^{N+1}}}(|\Uparrow\rangle + |\Downarrow\rangle) \otimes_{k=1}^{N} \left(|\uparrow\rangle + e^{i\phi_k}|\downarrow\rangle\right)$$

Spin echo simulations



N = 14, all curves averaged over 100 OU processes

Quantum quenches

Quenches
(more generally: 'prepare and release')

The problem: considering a generic initial state, what is the time evolution of the system?

Hamiltonian driving time evolution

$$|\Psi(t)\rangle = e^{-iHt}|\Psi(t=0)\rangle$$

initial state is NOT eigenstate of H

Observable expectation values depend on time:

$$\bar{\mathcal{O}}(t) \equiv \frac{\langle\Psi(t)|\mathcal{O}|\Psi(t)\rangle}{\langle\Psi(t)|\Psi(t)\rangle}$$

First question: what is the steady state long after the quench?

Fundamental issue: does the system relax? thermalize?

Crucial point: *time evolution in the presence of myriads of constraints (due to integrability) is special*

Conjecture: *steady state is described by a generalized Gibbs ensemble (GGE)*

$$\lim_{t\to\infty} \bar{\mathcal{O}}(t) = \langle \mathcal{O} \rangle_{GGE} = \frac{\mathrm{Tr}\{\mathcal{O} e^{-\sum_n \beta_n Q_n}\}}{\mathrm{Tr}\{e^{-\sum_n \beta_n Q_n}\}}$$

Rigol, Dunjko, Yurovsky, Olshanii, PRL 2007

see also Jaynes, Phys. Rev. 1957

GGE implementation

Generalized inverse temperatures to be set using the initial conditions on conserved charges

$$\langle \hat{Q}_m \rangle = \mathrm{Tr}\left\{ \hat{Q}_m e^{-\sum_n \beta_n \hat{Q}_n} \right\} / \mathcal{Z}_{GGE} \quad m = 0, 1, 2, \ldots$$

where $\mathcal{Z}_{GGE} = \mathrm{Tr} e^{-\sum_n \beta_n \hat{Q}_n}$

In reality, two major difficulties:

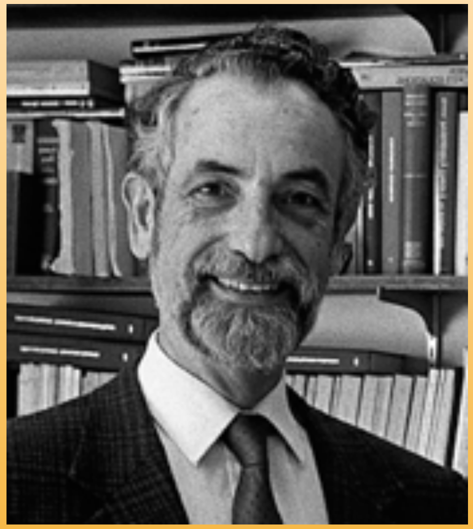
- Conserved charges are generically nontrivial
- Self-consistency problem difficult to solve

In practice: implementable for free theories only (charges: momentum occupation modes)

For interacting cases: not understood in general.

Quantum quenches:

BEC to repulsive Lieb-Liniger quench



Interacting Bose gas (Lieb-Liniger)

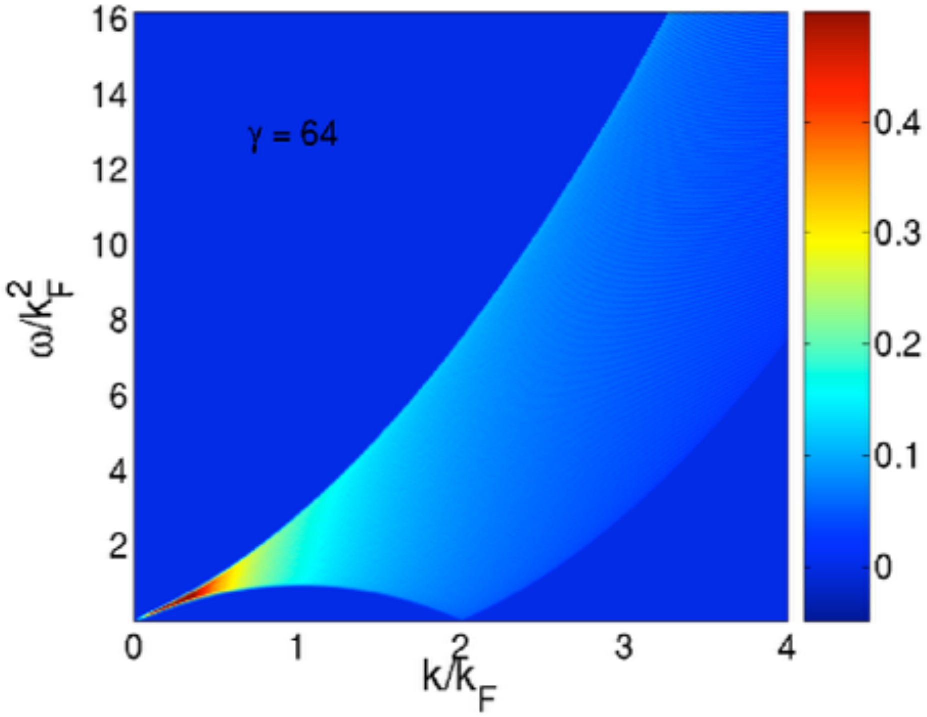
$$\mathcal{H}_N = -\sum_{j=1}^{N} \frac{\partial^2}{\partial x_j^2} + 2c \sum_{1 \leq j < l \leq N} \delta(x_j - x_l)$$

Exact eigenstates from Bethe Ansatz:

$$\Psi\left(\mathbf{x}|\boldsymbol{\lambda}\right) = F_\lambda \sum_{P \in S_N} A_P(\mathbf{x}|\boldsymbol{\lambda}) \prod_{j=1}^{N} e^{i\lambda_{P_j} x_j}$$

$$F_\lambda = \frac{\prod_{j>k=1}^{N}(\lambda_j - \lambda_k)}{\sqrt{N! \prod_{j>k=1}^{N}\left((\lambda_j - \lambda_k)^2 + c^2\right)}}$$

$$A_P(\mathbf{x}|\boldsymbol{\lambda}) = \prod_{j<k=1}^{N}\left(1 - \frac{ic\ \mathrm{sgn}(x_j - x_k)}{\lambda_{P_j} - \lambda_{P_k}}\right)$$

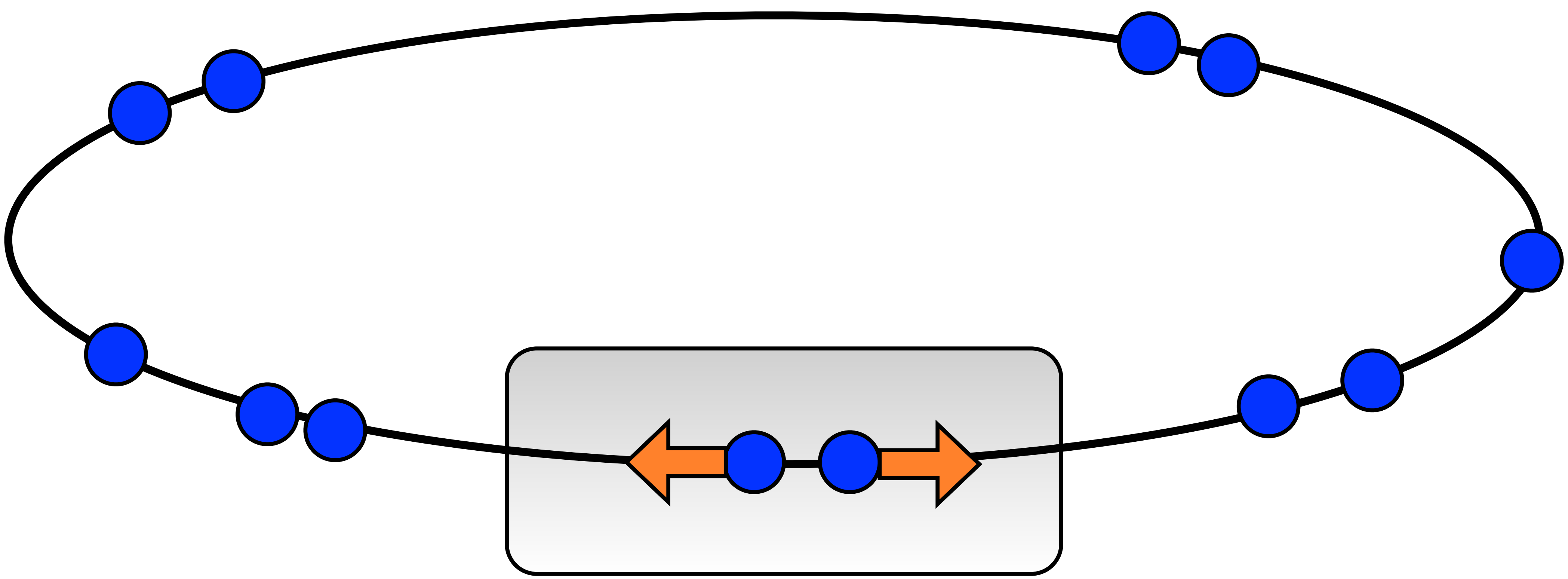


Quench from BEC to repulsive gas

Start from GS of noninteracting theory,

$$|0_N\rangle \equiv \frac{1}{\sqrt{L^N N!}} \left(\psi^\dagger_{k=0}\right)^N |0\rangle$$

Turn repulsive interactions on from t=0 onwards:



particles 'repel away' from each other,
system heats up, momentum distribution broadens, ...

This is a difficult problem to treat...

1) Generalized Gibbs ensemble logic

Kormos, Shashi, Chou and Imambekov, arxiv:1204.3889

Conserved charges:

$$\hat{Q}_n : \qquad \hat{Q}_n |\{\lambda\}_N\rangle = Q_n |\{\lambda\}_N\rangle$$

$$Q_n(\{\lambda\}_N) = \sum_{j=1}^{N} \lambda_j^n$$

Davies 1990; Davies and Korepin

→ GGE inapplicable, charges take infinite values!

J-S C + J. Mossel, unpublished

2) GGE on lattice, q-deformed model

→ Works, partial results only (using a few charges)

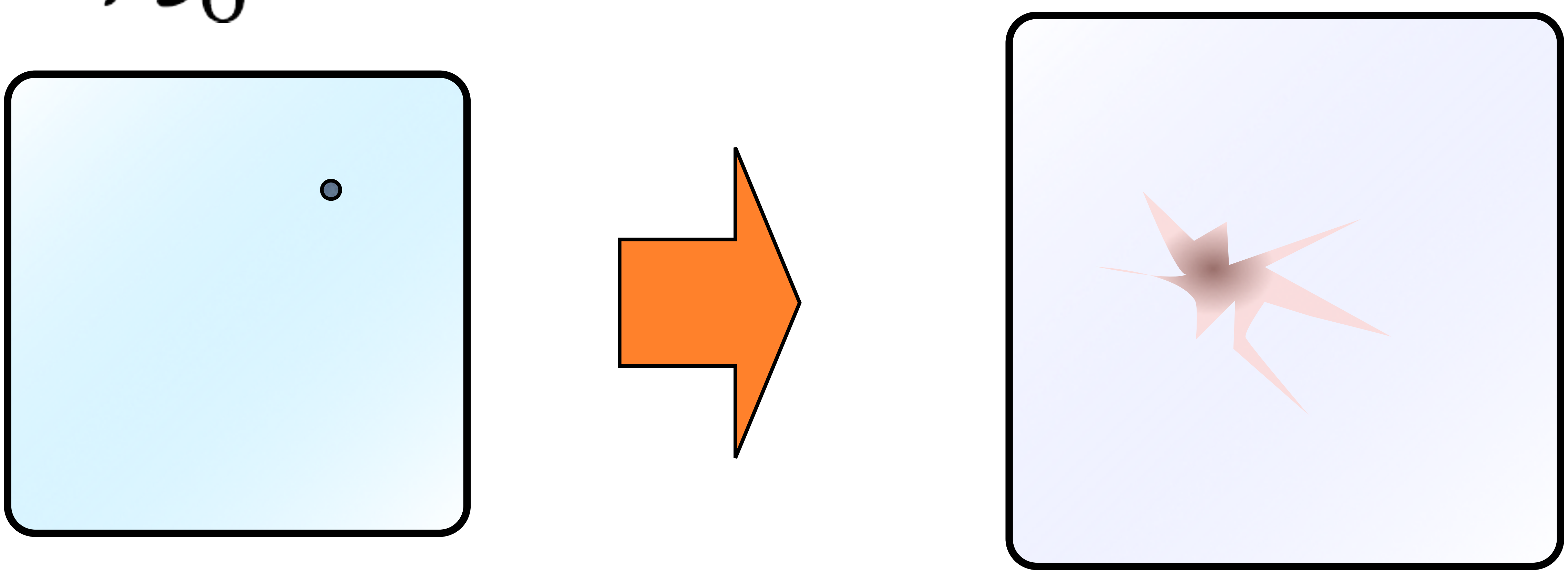
Kormos, Shashi, Chou, JSC, Imambekov, PRA 2014

The ‘quench action’ approach

J-SC & F.H.L. Essler, PRL 2013

in pictures...

$\mathcal{H}_0$ Initial state: $\mathcal{H}$



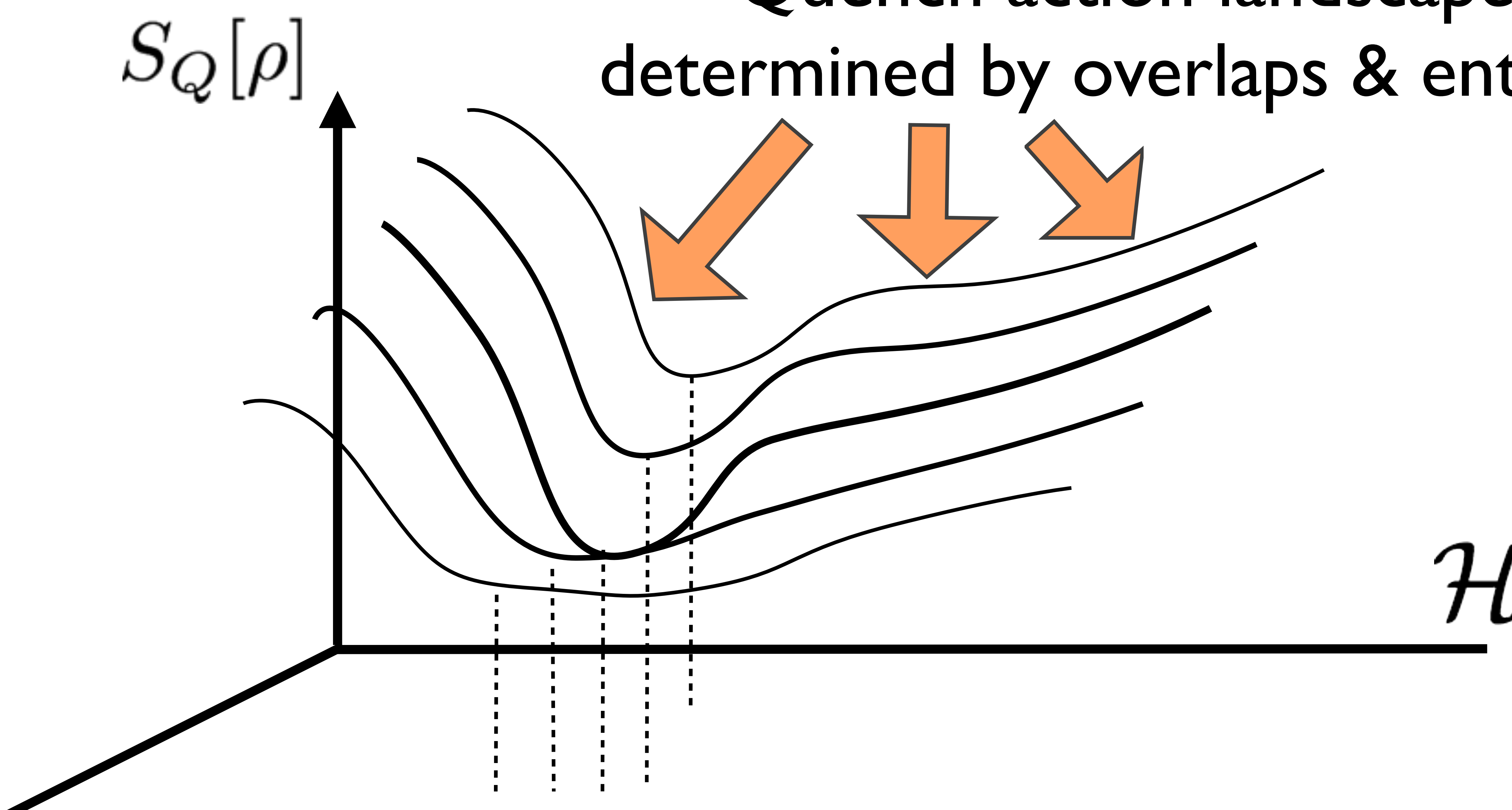
in pre-quench Hilbert space basis

in post-quench Hilbert space basis

The 'quench action' approach

J-SC & F.H.L. Essler, PRL 2013

Quench action landscape:
determined by overlaps & entropy

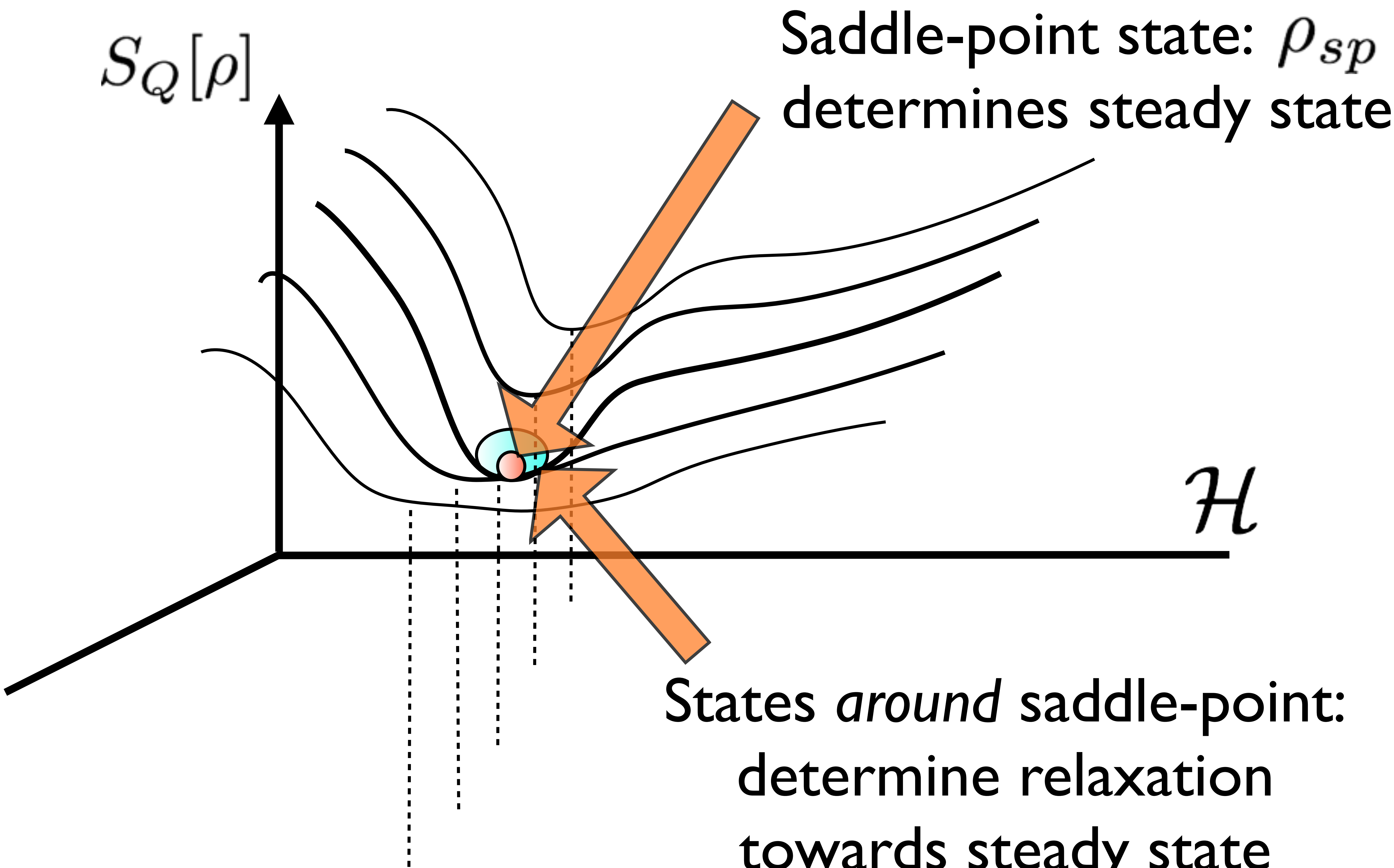


Variational approach, implemented by a
'Generalized thermodynamic Bethe Ansatz'

J. Mossel and J-SC, JPA 2012; J-SC & R. Konik, PRL 2012,
see also Fioretto & Mussardo NJP 2010, Pozsgay JSTAT 2011

The ‘quench action’ approach

J-SC & F.H.L. Essler, PRL 2013



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now in equations...

Consider a generic integrable model, with eigenstates labeled by quantum numbers $\{I\}$

Resolution of identity: $\mathbf{1} = \sum_{\{I\}} |\{I\}\rangle\langle\{I\}|$

Arbitrary initial state can be decomposed in this basis:

$$|\Psi(t=0)\rangle = \sum_{\{I\}} e^{-S^{\Psi}_{\{I\}}} |\{I\}\rangle$$

using overlap coefficients $S^{\Psi}_{\{I\}} = -\ln\langle\{I\}|\Psi(t=0)\rangle \in \mathbb{C}$

Time dependence: trivially written as

$$|\Psi(t)\rangle = \sum_{\{I\}} e^{-S^{\Psi}_{\{I\}} - i\omega_{\{I\}} t} |\{I\}\rangle$$

The expectation values we're interested in then become

$$\bar{\mathcal{O}}(t) = \frac{\sum_{\{I^l\}} \sum_{\{I^r\}} e^{-(S^{\Psi}_{\{I^l\}})^* - S^{\Psi}_{\{I^r\}} + i(\omega_{\{I^l\}} - \omega_{\{I^r\}})t} \langle \{I^l\} | O | \{I^r\} \rangle}{\sum_{\{I\}} e^{-2\Re S^{\Psi}_{\{I\}}}}$$

Intractable in general: double Hilbert space sum

Need to develop tools to evaluate this...

Start by looking at wavefunction normalization:

$$\langle \Psi(t)|\Psi(t)\rangle = \sum_{\{I\}} e^{-2\Re S^{\Psi}_{\{I\}}}$$

In Th.Lim., would like to use the usual functional integral

$$\lim_{Th} \sum_{\{I\}} (\ldots) = \int D\rho \; e^{S^{YY}[\rho]} (\ldots)$$

Including the effective overlaps yields

$$\lim_{Th} \langle \Psi(t)|\Psi(t)\rangle = \int D\rho \; e^{-S^{Q}[\rho]}$$

with 'quench action'

$$S^{Q}[\rho] = S^{o}[\rho] - S^{YY}[\rho]$$

Saddle-point evaluation:

$$\rho_{sp} : \quad \text{such that} \quad \left. \frac{\delta S^{Q}[\rho]}{\delta \rho} \right|_{\rho_{sp}} = 0$$

→ 'Generalized thermodynamic Bethe Ansatz'

For operator expectation values, we had

$$\bar{\mathcal{O}}(t) = \frac{\sum_{\{I^l\}} \sum_{\{I^r\}} e^{-(S^{\Psi}_{\{I^l\}})^* - S^{\Psi}_{\{I^r\}} + i(\omega_{\{I^l\}} - \omega_{\{I^r\}})t} \langle \{I^l\}|O|\{I^r\}\rangle}{\sum_{\{I\}} e^{-2\Re S^{\Psi}_{\{I\}}}}$$

Considering operators which are 'weak'
(creating a non-entropically large nr of excitations when acting on a given state): thermodyn limit is

$$\lim_{Th} \bar{\mathcal{O}}(t) = \frac{\int \mathcal{D}\rho e^{-S^Q[\rho]} \ \lim_{Th} \sum_{\{\mathbf{e}\}} e^{-\delta S_{\{\mathbf{e}\}}[\rho] - i\omega_{\{\mathbf{e}\}}[\rho]t} \langle \rho|\mathcal{O}|\rho; \{\mathbf{e}\}\rangle}{\int \mathcal{D}\rho \ e^{-S^Q[\rho]}}$$

relative overlaps

denumerable set of excitations

For operators with non-entropically large matrix elements, can perform a saddle-point evaluation (same saddle-point in numerator and denominator)

$$\lim_{Th} \bar{\mathcal{O}}(t) = \lim_{Th} \frac{1}{2} \sum_{\{\mathbf{e}\}} \Big[e^{-\delta S_{\{\mathbf{e}\}}[\rho_{sp}] - i\omega_{\{\mathbf{e}\}}[\rho_{sp}]t} \langle \rho_{sp} | \mathcal{O} | \rho_{sp}; \{\mathbf{e}\} \rangle + e^{-\delta S^*_{\{\mathbf{e}\}}[\rho_{sp}] + i\omega_{\{\mathbf{e}\}}[\rho_{sp}]t} \langle \rho_{sp}; \{\mathbf{e}\} | \mathcal{O} | \rho_{sp} \rangle \Big]$$

Main message: the *full* time dependence is recoverable using a minimal amount of data

- saddle-point distribution (from GTBA)
- excitations in vicinity of sp state (easy)
- differential overlaps
- selected matrix elements

For operators with non-entropically large matrix elements, can perform a saddle-point evaluation (same saddle-point in numerator and denominator)

$$\lim_{Th} \bar{\mathcal{O}}(t) = \lim_{Th} \frac{1}{2} \sum_{\{\mathbf{e}\}} \Big[e^{-\delta S_{\{\mathbf{e}\}}[\rho_{sp}] - i\omega_{\{\mathbf{e}\}}[\rho_{sp}]t} \langle \rho_{sp} | \mathcal{O} | \rho_{sp}; \{\mathbf{e}\} \rangle + e^{-\delta S^*_{\{\mathbf{e}\}}[\rho_{sp}] + i\omega_{\{\mathbf{e}\}}[\rho_{sp}]t} \langle \rho_{sp}; \{\mathbf{e}\} | \mathcal{O} | \rho_{sp} \rangle \Big]$$

Main message: the *full* time dependence is recoverable using a minimal amount of data

Large time limit:

$$\lim_{t\to\infty} \lim_{Th} \bar{\mathcal{O}}(t) = \lim_{Th} \langle \rho_{sp} | \mathcal{O} | \rho_{sp} \rangle$$

Single representative state is sufficient to calculate properties of steady state! (think of ETH)

Cassidy, Rigol PRL 2009

Back to BEC-LL quench

Problem: need to calculate overlaps.

Remark: only parity-invariant states contribute since

$$0 = \langle 0|\hat{Q}_{2m+1}|I\rangle = \langle 0|I\rangle \sum_{j=1}^{N} \lambda_j^{2m+1}$$

Known large c limit: Gritsev, Rostunov & Demler, JSTAT 2010

$$\langle \{\lambda_j\}_{j=1}^{N/2}, \{-\lambda_j\}_{j=1}^{N/2}|0\rangle \propto \prod_{\lambda_j>0} \frac{1}{\lambda_j}$$

Possible for generic interaction?

Back to BEC-LL quench

Explicit result:

J. De Nardis, B. Wouters, M. Brockmann & J-SC, PRA 89, 2014

M. Brockmann JPA 2014

$$\langle \{\lambda_j\}_{j=1}^{N/2}, \{-\lambda_j\}_{j=1}^{N/2} | 0 \rangle = \sqrt{\frac{(cL)^{-N} N!}{\det_{j,k=1}^{N} G_{jk}}} \frac{\det_{j,k=1}^{N/2} G_{jk}^{Q}}{\prod_{j=1}^{N/2} \frac{\lambda_j}{c} \sqrt{\frac{\lambda_j^2}{c^2} + \frac{1}{4}}}$$

(reminiscent of Gaudin formula)

with matrix $$G_{jk}^{Q} = \delta_{jk} \Big(L + \sum_{l=1}^{N/2} K^{Q}(l_j, l_l) \Big) - K^{Q}(l_j, l_k)$$

$$K^{Q}(\lambda, \mu) = K(\lambda - \mu) + K(\lambda + \mu) \qquad K(\lambda) = \frac{2c}{\lambda^2 + c^2}$$

Quench action approach to BEC-LL quench

J. De Nardis, B. Wouters, M. Brockmann & J-SC, PRA 89, 2014

We are now in position to apply the quench action logic!

Need thermodynamic limit form of overlaps:

$$\lim_{Th} \langle \lambda, -\lambda | 0 \rangle = \exp\left(-\frac{L}{2} n \left(\log \frac{c}{n} + 1 \right) \right) \times \exp\left\{ -\frac{L}{2} \int_0^\infty d\lambda \rho(\lambda) \log \left[\frac{\lambda^2}{c^2} \left(\frac{\lambda^2}{c^2} + \frac{1}{4} \right) \right] + \mathcal{O}(L^0) \right\}$$

Quench action now defined, saddle-point solution via generalized thermodynamic Bethe ansatz

Quench action solution to BEC-LL quench

J. De Nardis, B. Wouters, M. Brockmann & J-SC, PRA 89, 2014

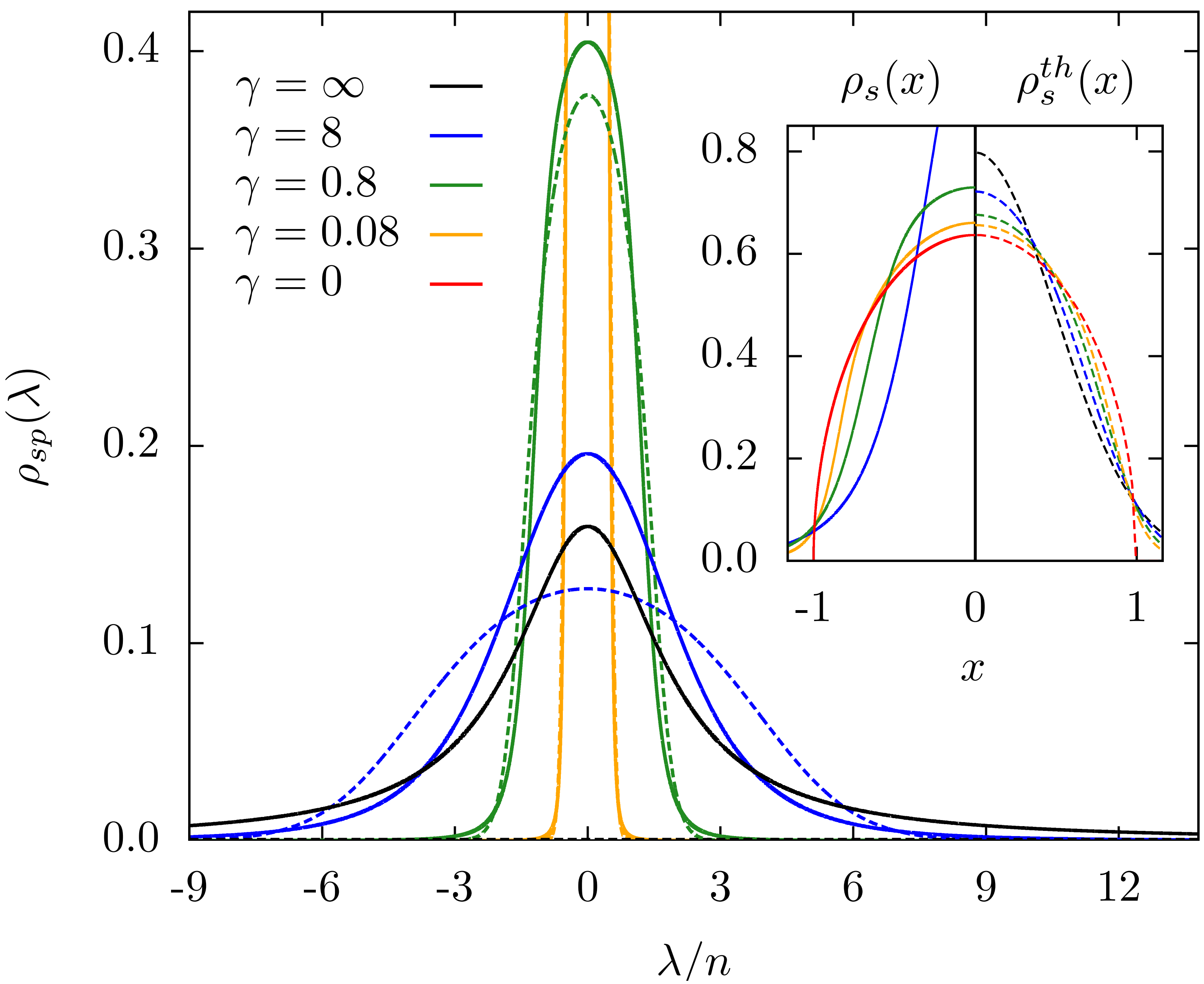
It is in fact possible to give a closed form solution of the GTBA for the saddle-point state, for any value of the interaction:

$$\rho(\lambda) = -\frac{\gamma}{2\pi}\frac{\partial a(\lambda)}{\partial \gamma}(1 + a(\lambda))^{-1}$$

$$a(\lambda) = \frac{2\pi/\gamma}{\frac{\lambda}{c}\sinh\left(\frac{2\pi\lambda}{c}\right)} I_{1-2i\frac{\lambda}{c}}\left(\frac{4}{\sqrt{\gamma}}\right) I_{1+2i\frac{\lambda}{c}}\left(\frac{4}{\sqrt{\gamma}}\right)$$

Quench action solution to BEC-LL quench

J. De Nardis, B. Wouters, M. Brockmann & J-SC, PRA 89, 2014



Subplot: scaled fn

$$\rho_s(x) = \sqrt{\gamma}\rho(c\sqrt{\gamma}x/2)/2$$

Large c:

$$\rho(\lambda) = \frac{1}{2\pi}\frac{4n^2}{\lambda^2 + 4n^2}$$

Small c: semicircle

$$\rho(\lambda) \sim \frac{1}{\pi\sqrt{\gamma}}\sqrt{1 - \frac{\lambda^2}{4\gamma n^2}}$$

Asymptotics as from q-bosons: $2\pi\rho(\lambda) \sim \frac{n^4\gamma^2}{\lambda^4} + \frac{n^6\gamma^3(24-\gamma)}{4\lambda^6} + \ldots$

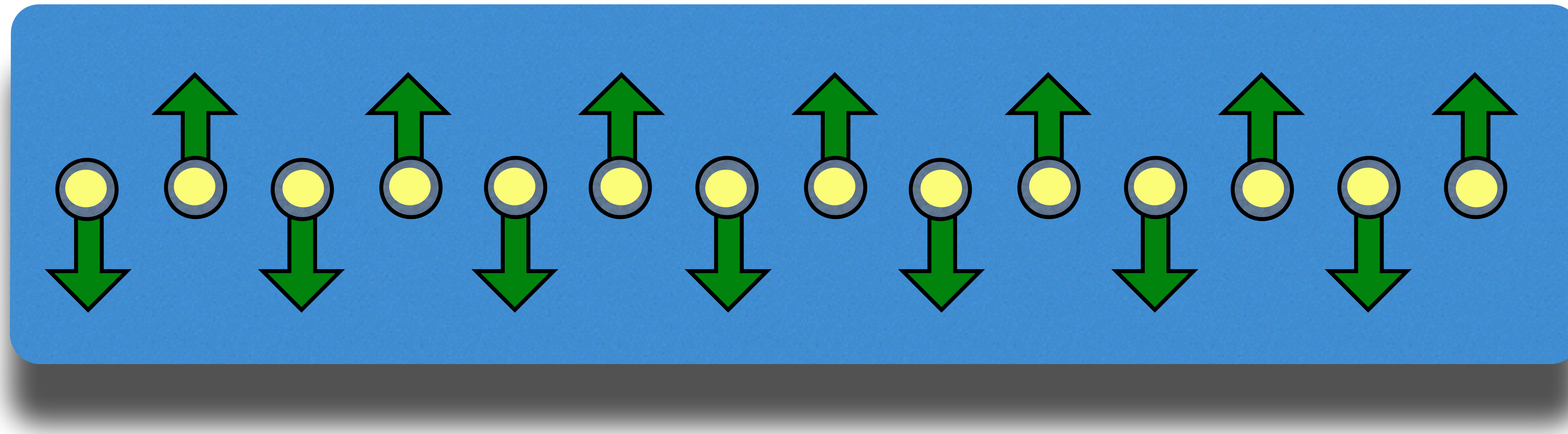
Tail explains divergences of evalues of conserved charges

Quantum quenches:

Néel to XXZ quench

Quench from Néel to XXZ

Start from Néel state:



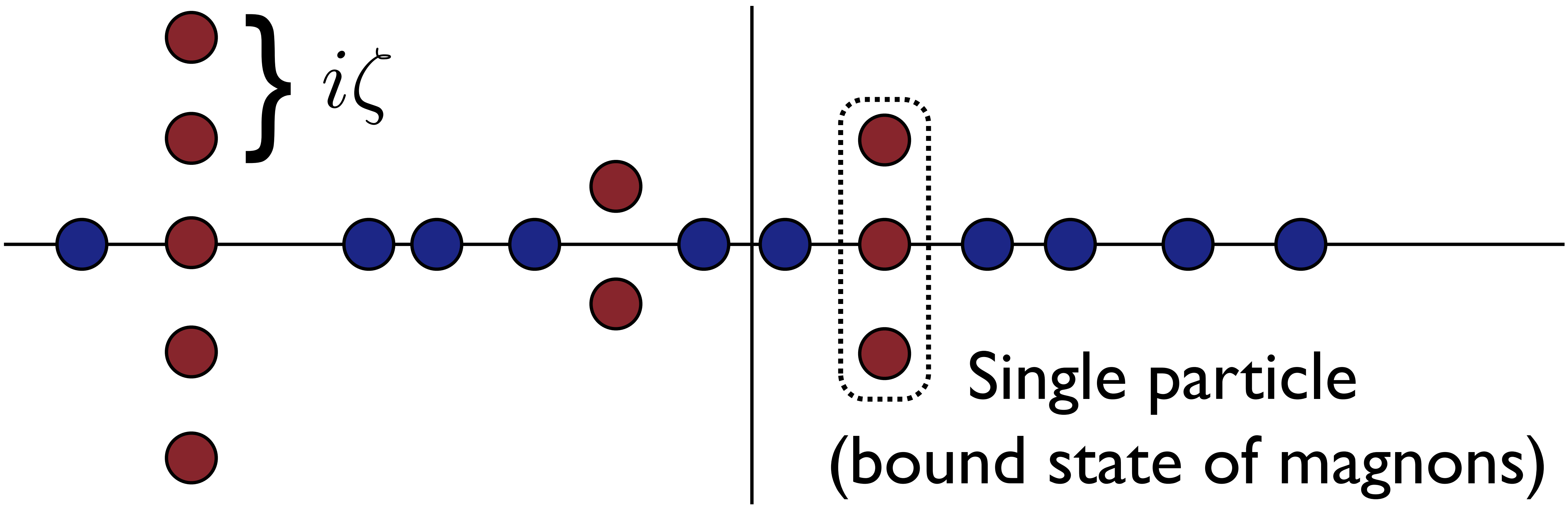
From t=0 onwards, evolve with XXZ Hamiltonian

$$H = \sum_{j=1}^{N} \left[J(S_j^x S_{j+1}^x + S_j^y S_{j+1}^y + \Delta S_j^z S_{j+1}^z) - H_z S_j^z \right]$$

Can one treat this problem exactly?

"Particle content" of XXZ: nontrivial

Solution of Bethe equations: rapidities + strings



$$\lambda_\alpha^{j,a} = \lambda_\alpha^j + i\frac{\zeta}{2}(n_j + 1 - 2a) + i\delta_\alpha^{j,a}$$

$\delta_\alpha^{j,a} \sim \mathrm{O}(e^{-(cst)N})$

Classification of strings: Bethe, Takahashi, Suzuki, ...

Quench action approach to Néel-XXZ quench

First step: exact overlaps of Néel state with XXZ eigenstates

Tsuchiya JMP1998; Kozlowski & Pozsgay JSTAT 2012; Pozsgay arxiv 2013

(Gaudin-like form again!)

M. Brockmann, J. De Nardis, B. Wouters & J-SC JPA 2014

See also talk of M. Brockmann

$$\frac{\langle\Psi_0|\{\pm\lambda_j\}_{j=1}^{M/2}\rangle}{\|\{\pm\lambda_j\}_{j=1}^{M/2}\|} = \sqrt{2}\left[\prod_{j=1}^{M/2}\frac{\sqrt{\tan(\lambda_j+i\eta/2)\tan(\lambda_j-i\eta/2)}}{2\sin(2\lambda_j)}\right]\sqrt{\frac{\det_{M/2}(G^+_{jk})}{\det_{M/2}(G^-_{jk})}}$$

$$G^{\pm}_{jk} = \delta_{jk}\left(NK_{\eta/2}(\lambda_j) - \sum_{l=1}^{M/2}K^+_\eta(\lambda_j,\lambda_l)\right) + K^{\pm}_\eta(\lambda_j,\lambda_k)$$

$$K^{\pm}_\eta(\lambda,\mu) = K_\eta(\lambda-\mu) \pm K_\eta(\lambda+\mu) \qquad K_\eta(\lambda) = \frac{\sinh(2\eta)}{\sin(\lambda+i\eta)\sin(\lambda-i\eta)}$$

Quench action approach to Néel-XXZ quench

Second step: generalized TBA

B. Wouters, J. De Nardis, M. Brockmann, D. Fioretto, M.Rigol & J-SC, PRL 2014

$$\ln \eta_n(\lambda) = -2\,h\,n - \ln W_n(\lambda) + \sum_{m=1}^{\infty} a_{nm} * \ln\left(1+\eta_m^{-1}\right)(\lambda)$$

where $\eta_n(\lambda) \equiv \rho_{n,h}(\lambda)/\rho_n(\lambda)$ $\quad a_n(\lambda) = \frac{1}{\pi}\frac{\sin n\eta}{\cosh n\eta - \cos 2\lambda}$

and the effective driving terms (pseudo-energies) are

$$W_n(\lambda) = \begin{cases} \frac{1}{2^{n+1}\sin^2 2\lambda}\frac{\cosh n\eta - \cos 2\lambda}{\cosh n\eta + \cos 2\lambda}\prod_{j=1}^{\frac{n-1}{2}}\left(\frac{\cosh(2j-1)\eta - \cos 2\lambda}{(\cosh(2j-1)\eta + \cos 2\lambda)(\cosh 4\eta j - \cos 4\lambda)}\right)^2 & \text{if } n \text{ odd,} \\ \frac{\tan^2\lambda}{2^n}\frac{\cosh n\eta - \cos 2\lambda}{\cosh n\eta + \cos 2\lambda}\frac{1}{\prod_{j=1}^{\frac{n}{2}}(\cosh 2(2j-1)\eta - \cos 4\lambda)^2}\prod_{j=1}^{\frac{n-2}{2}}\left(\frac{\cosh 2j\eta - \cos 2\lambda}{\cosh 2j\eta + \cos 2\lambda}\right)^2 & \text{if } n \text{ even.} \end{cases}$$

Solution of this GTBA gives steady-state

(analytically! See M. Brockmann's talk)

Quench action approach to Néel-XXZ quench

Equivalent form of generalized TBA:

$$\ln(\eta_n) = d_n + s * \left[\ln(1+\eta_{n-1}) + \ln(1+\eta_{n+1}) \right]$$

with driving terms $d_n(\lambda) = \sum_{k\in\mathbb{Z}} e^{-2ik\lambda} \frac{\tanh(\eta k)}{k} \left((-1)^n - (-1)^k\right)$

GGE with local charges: same form of coupled equations, but driving term only for n=1:

$$d_1(\lambda) = -\frac{1}{\pi} \sum_{m=1}^{\infty} \beta_{2m} \sum_{k\in\mathbb{Z}} e^{-2ik\lambda} \frac{k^{2m-2}}{\cosh k\eta}$$

unknown

Néel-XXZ quench: conserved charges

Initial expectation value of local charges:

Fagotti & Essler JSTAT 2013

$$\lim_{N\to\infty}\frac{1}{N}\langle\text{Néel}|Q_{n+1}|\text{Néel}\rangle = -\frac{\Delta}{2}\frac{\partial^{n-1}}{\partial x^{n-1}}\frac{1-\Delta^2}{\cosh[\sqrt{1-\Delta^2}x]-\Delta^2}\Bigg|_{x=0}$$

In 1-to-1 correspondence with 1-string hole density:

B. Wouters, J. De Nardis, M. Brockmann, D. Fioretto, M. Rigol & J-SC, PRL 2014

$$\sum_{k\in\mathbb{Z}} k^{2m-2}\left(\frac{e^{-|k|\eta}-\hat{\rho}_1^h(k)}{2\cosh k\eta}\right) = \langle Q_{2m}\rangle \quad m\in\mathbb{N}$$

which fixes

$$\rho_{1,h}^{\text{Néel}}(\lambda) = \frac{\pi^2 a_1^3(\lambda)\sin^2(2\lambda)}{\pi^2 a_1^2(\lambda)\sin^2(2\lambda)+\cosh^2(\eta)}$$

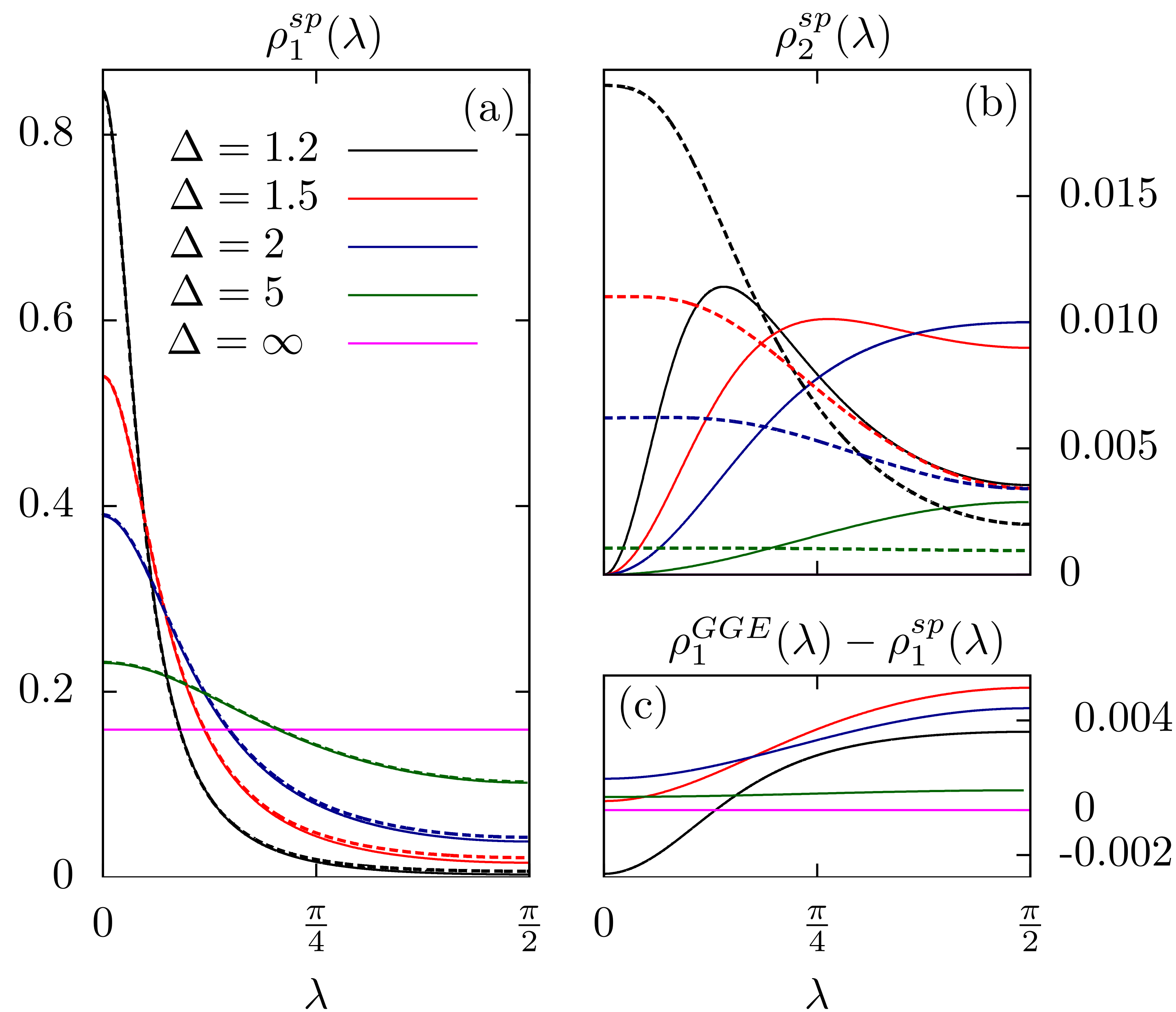
Quench action nontrivially reproduces this;
GGE also of course, but only by definition

The steady state: Néel to XXZ

Solid lines:
quench action

Dashed lines:
GGE (local charges)

QA and GGE have different saddle-point densities

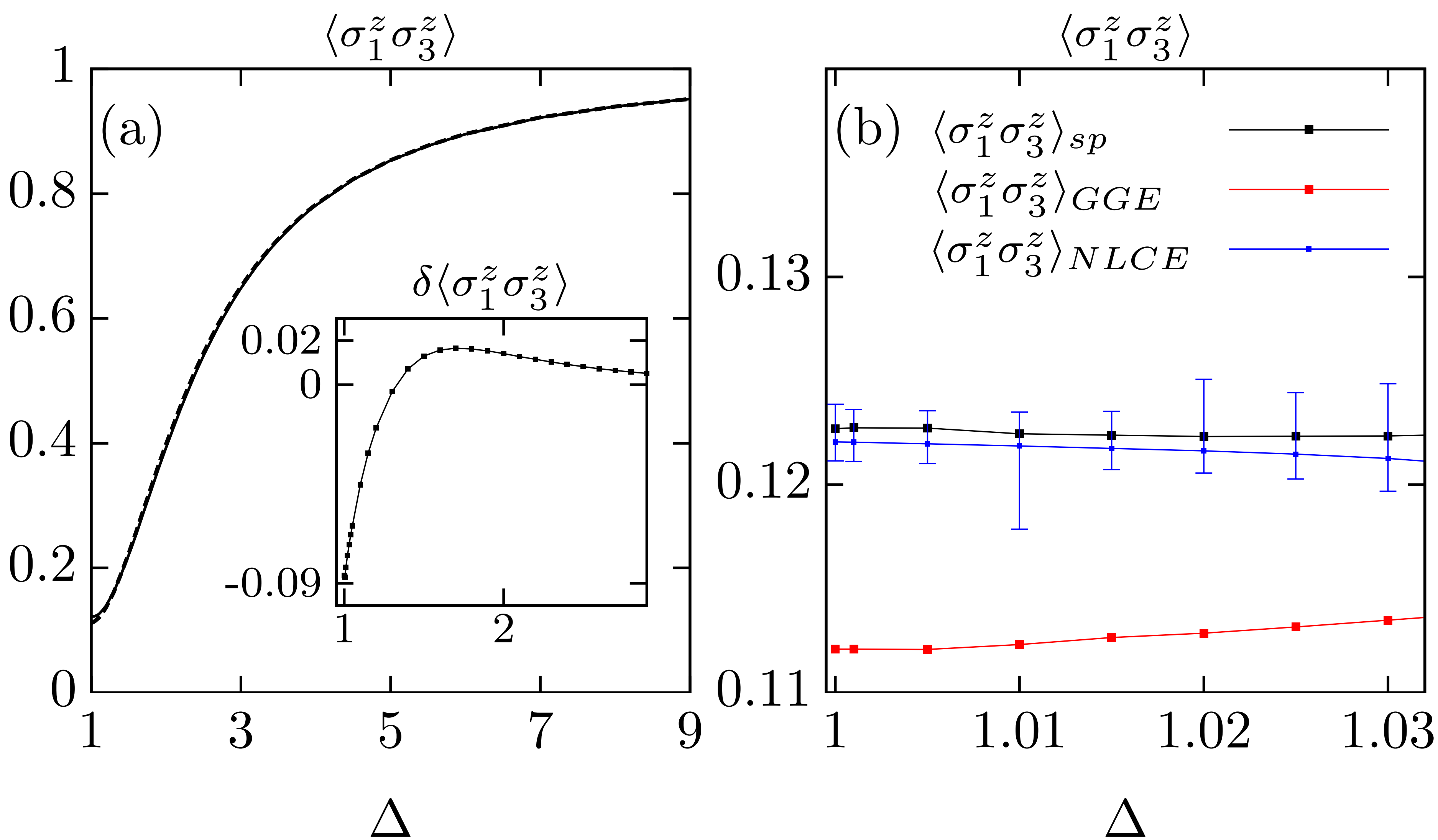


Large Delta expansion:

$$\rho_1^{GGE} - \rho_1^{sp} = \frac{1}{4\pi\Delta^2} + O(\Delta^{-3}),$$

$$\rho_2^{GGE} - \rho_2^{sp} = \frac{1 - 3\sin^2(\lambda)}{3\pi\Delta^2} + O(\Delta^{-3}).$$

Difference in distribution: impact on correlations



Numerical verification using NLCE (M. Rigol)

Large Delta expansions:

$$\langle\sigma_1^z\sigma_2^z\rangle_{QA} = -1 + \frac{2}{\Delta^2} - \frac{7}{2\Delta^4} + \frac{77}{16\Delta^6} + \dots$$

$$\langle\sigma_1^z\sigma_2^z\rangle_{GGE} = -1 + \frac{2}{\Delta^2} - \frac{7}{2\Delta^4} + \frac{43}{8\Delta^6} + \dots$$

Not convinced?

Look at other results by Budapest group

B. Pozsgay, M. Mestyán, M.A. Werner, M. Kormos, G. Zaránd, G. Takács, arxiv 1405.2843

- reobtain our Néel results
- also consider initial dimer state
- obtain numerical (iTEBD) evidence for correlations being different in dimer case

There remains little doubt about the correctness of the quench action results

See also recent results by G. Goldstein & N. Andrei

What's going on?

Néel to XXZ: current situation

- *quench action solution gives correct expectation value for all conserved charges, directly from microscopics*
- *quench action and (local) GGE steady state distributions do not coincide*
- *these different distributions lead to different observable expectation values*

Possible explanations of this mismatch:

- *GGE converges to QA once all (nonlocal*) charges are added*
- *exceptional states invalidate the QA calculation* ruled out

* *of which there are exponentially many more than local ones!*

Summary & perspectives

- Integrability for dynamics
  - *prototypical strong correlations under control*
  - *detailed experimental correspondence in spin chains*
  - *…and now also in cold atomic Bose gases*
  - *useful tests and lessons for field theory/numerical methods*

- Quench action logic
  - *new approach to out-of-equilibrium problems*
  - *gives access to full time evolution with minimal data*
  - *BEC to LL: exact solution (inaccessible to GGE)*
  - *Néel to XXZ: exact solution*
  - *GGE with local charges gives different steady state!*