Abstract
We derive expressions for the form factors of the quantum transfer matrix of the spin-1/2 XXZ chain which allow us to take the infinite Trotter number limit. This solves the longstanding problem of describing analytically the amplitudes in the leading asymptotics of the finite temperature correlation functions of the model. In the zero-temperature limit, we recover the predictions of conformal field theory (CFT) and Luttinger liquid (LL) approach concerning the large-distance behaviour of the correlators in the massless phase. As a first result for the massive regime, we obtain Baxter’s spontaneous magnetization.

Introduction
The Hamiltonian of the XXZ chain in a longitudinal magnetic field $h$ is given by

$$H = J \sum_{j=1}^{L} \left( \sigma_j^x \sigma_{j+1}^x + \sigma_j^y \sigma_{j+1}^y + \Delta \sigma_j^z \sigma_{j+1}^z \right) - \frac{2}{L} \sum_{j=1}^{L} \sigma_j^z,$$

where periodic boundary conditions are implied. $J > 0$ fixes the energy scale and $\Delta = \text{ch}(\eta) \in \mathbb{R}$ is the anisotropy parameter. We investigate the large-distance asymptotics of longitudinal and transversal correlation functions in the thermodynamic limit $L \to \infty$ at finite temperature $T$.

Quantum transfer matrix and Bethe ansatz
We study finite temperature properties of the system by means of the quantum transfer matrix (QTM) $t(\lambda)$ which is defined as the column-to-column transfer matrix of a certain inhomogeneous six-vertex model.

The monodromy matrix $T(\lambda)$ is a $2 \times 2$ matrix with matrix elements $A(\lambda), B(\lambda), C(\lambda), D(\lambda) \in \text{End}(C^2)$.

The QTM $t(\lambda) = \text{Tr} \left( \bar{T}(\lambda) T(\lambda) \right)$ can be diagonalized using the algebraic Bethe ansatz (ABA).

Eigenvectors have the form $|\psi\rangle = B(\lambda_0) \ldots B(\lambda_0) |0\rangle$ where the Bethe roots $\{\lambda_j\}_j$ have to satisfy the Bethe ansatz equations (BAE).

The QTM $|\psi\rangle$ has a unique eigenvalue $A(0)$ with largest modulus which, together with the corresponding eigenvector $|\Psi\rangle$, determines the state of thermal equilibrium completely for $L \to \infty$.

Therefore, the low-T analysis requires the following steps:

1. Calculation of amplitudes and correlation lengths for small but finite $T$
2. Summation of the form factor series with the formula of [Kitanine et al. 11]

Result:
$$\langle \sigma_i \sigma_j \rangle \sim \left( 1 - \frac{1}{\eta} \right)^{\frac{\eta}{2}} A_{\eta} \beta_0 \frac{\pi T}{h v F} \frac{1}{\sinh(\pi T/h v F)} \frac{1}{\sinh(\pi T/h v F)}$$

Amplitudes for longitudinal correlators
Our main result is the following analytic expression for the amplitudes in the Trotter limit:

$$A_{\eta}(\lambda) = \tau \exp \left( \int_{\lambda_0}^{\lambda} \frac{d\lambda'}{2\pi i \eta} \frac{\pi T}{h v F} \frac{1}{\sinh(\pi T/h v F)} \right) \frac{\eta}{2} \int_{\lambda_0}^{\lambda} \frac{d\lambda'}{2\pi i \eta} \frac{\pi T}{h v F} \frac{1}{\sinh(\pi T/h v F)} \left( 1 - A_{\eta}(\lambda') \right) \left( 1 - A_{\eta}(\lambda') \right) \left( 1 - A_{\eta}(\lambda') \right) \left( 1 - A_{\eta}(\lambda') \right) \left( 1 - A_{\eta}(\lambda') \right).$$

The determinants have to be understood as Fredholm determinants, e.g.

$$\det_{\lambda_0} \left( \begin{array}{c} 1 - R_+ \cr \eta \lambda - 1 - K \end{array} \right) = \int_{\lambda_0}^{\lambda} \frac{d\lambda'}{2\pi i \eta} \frac{\pi T}{h v F} \frac{1}{\sinh(\pi T/h v F)} \left( 1 - A_{\eta}(\lambda') \right) \left( 1 - A_{\eta}(\lambda') \right) \left( 1 - A_{\eta}(\lambda') \right) \left( 1 - A_{\eta}(\lambda') \right) \left( 1 - A_{\eta}(\lambda') \right).$$

Massless regime ($h_0 < h < h_0^c$)
In this regime, infinitely many terms contribute to the form factor series, which implies that the individual amplitudes must vanish as $T \to 0$. Therefore, the low-T analysis requires the following steps:

1. Calculation of amplitudes and correlation lengths for small but finite $T$
2. Summation of the form factor series with the formula of [Kitanine et al. 11]

Result:
$$\langle \sigma_i \sigma_j \rangle \sim \left( 1 - \frac{1}{\eta} \right)^{\frac{\eta}{2}} A_{\eta} \beta_0 \frac{\pi T}{h v F} \frac{1}{\sinh(\pi T/h v F)} \frac{1}{\sinh(\pi T/h v F)}$$

Amplitudes for transversal correlators
The expressions for the amplitudes pertaining to the transversal correlation functions have a remarkably similar structure:

$$A_{\eta}^{\perp}(\xi) = \tau \exp \left( \int_{0}^{\xi} \frac{d\xi'}{2\pi i \eta} \frac{\pi T}{h v F} \frac{1}{\sinh(\pi T/h v F)} \right) \frac{\eta}{2} \int_{0}^{\xi} \frac{d\xi'}{2\pi i \eta} \frac{\pi T}{h v F} \frac{1}{\sinh(\pi T/h v F)} \left( 1 - A_{\eta}^{\perp}(\xi') \right) \left( 1 - A_{\eta}^{\perp}(\xi') \right) \left( 1 - A_{\eta}^{\perp}(\xi') \right) \left( 1 - A_{\eta}^{\perp}(\xi') \right) \left( 1 - A_{\eta}^{\perp}(\xi') \right).$$

Zero-temperature limit for $\Delta > 1$

Form factor expansion
For the longitudinal two-point functions we start with a generating function with $\eta$ close to related to the twisted QTM $t(\lambda)$, which is given by

$$\left( q \sigma_i \sigma_j \right) = \left( q \sigma_i \sigma_j \right) + \left( q \sigma_i \sigma_j \right) + \left( q \sigma_i \sigma_j \right).$$

Thermal correlation functions can be expanded into series of form factors of the QTM. Instead of form factors of local operators, those of ABA operators appear. For $T > 0$ only a few terms contribute to the form factor expansion and correlators decay exponentially.

Correlation lengths $\xi$ have been studied extensively [Klümper et al. '01], so far little was known about the amplitudes $A_{\eta}$.