Quench echo and work statistics in integrable quantum field theories

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a joint work with Spyros Sotiriadis
arXiv:1403.7450 [cond-mat.stat-mech]

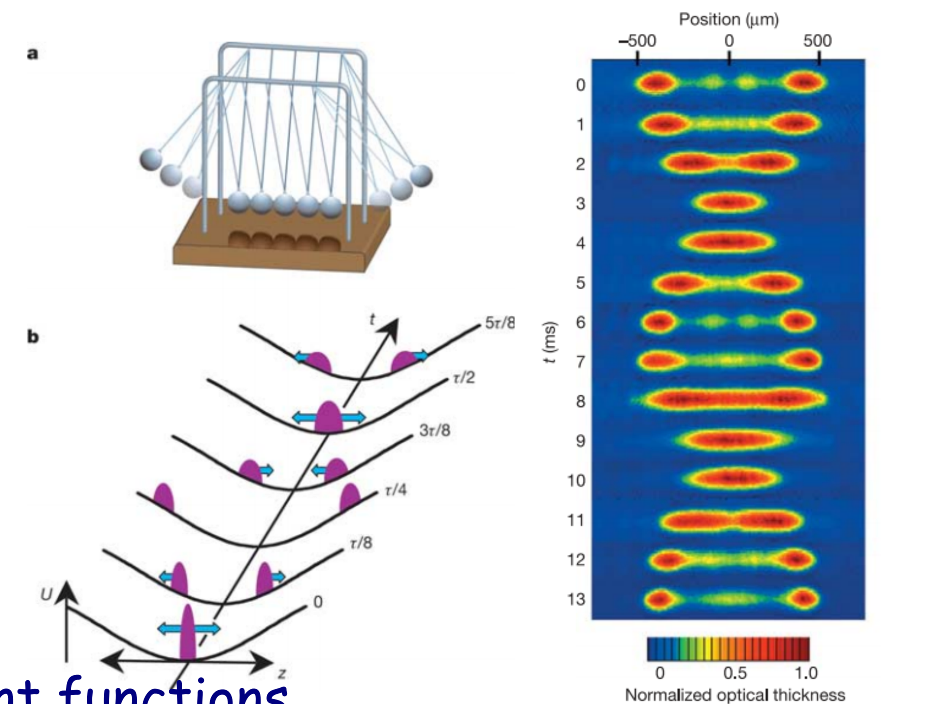
Quantum quenches

Quantum quench: paradigmatic out of equilibrium protocol

H_0 prepares ground state → H evolves system

Experimental progress in ultra cold atoms

- Coherent control, long coherence times
- Can realize paradigmatic model systems (Hubbard, Lieb-Liniger)
- Has become a playground for out of equilibrium quantum physics



Kinoshita, Nature 440 (2006), 900

One usually looks at evolution of n-point functions

$$\langle O\rangle(t),\ \langle O_1(x)O_2(0)\rangle(t)$$

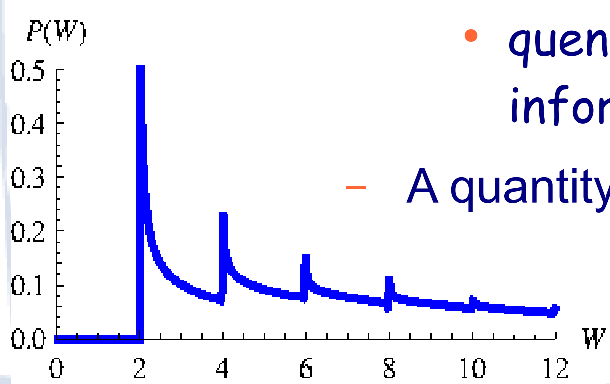
Thermalization? Role of integrability?

Quantum thermodynamics

- Treat the quench (or some more generic protocol) as a thermodynamic transformation: tdin variables: $\boxed{\text{work}}$, entropy, heat
- Why is this interesting?
  - e.g. 2nd law $W \geq \Delta F \rightarrow \langle W \rangle \geq \Delta F$
  - T ≠ 0: fluctuation relations
  - T = 0: in TDL: W is extensive and

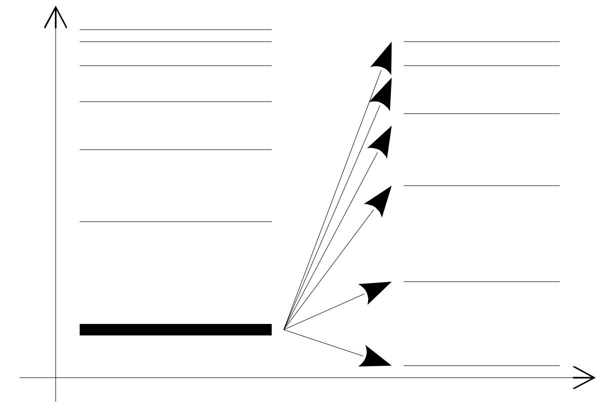
    $$P(W/V) \rightarrow \delta(\langle W/V \rangle)$$

    - interesting part: finite volume
    - quench and system specific information
  - A quantity “close to experiments”



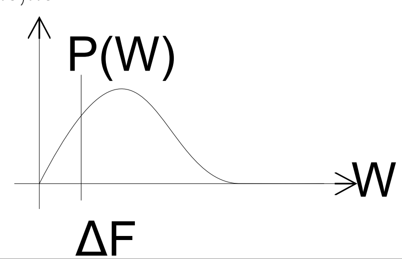
How to define work?

$$H_0 = \sum_n E_n |n\rangle\langle n| \quad H_1 = \sum_m \bar{E}_m |m\rangle\langle m|$$



- two projective energy meas
- for fast process stochactic

$$P(W) = \sum_{n,m} p_n p(m|n) \delta(W - (\bar{E}_m - E_n))$$



Results in low D systems

- Global: done so far:
  - Essentially free fermion systems (Ising, noisy Ising, Dicke)

    Silva (2008), Paraan-Silva (2009), Smacchia-Silva (2013), Marino-Silva (2014), Gambassi-Silva (2013)
  - Luttinger liquid Dora et al. (2012), (2013)
  - Free bosons (weak interaction limit of sine-Gordon, e.g. relative phase of two interacting BEC wires)

    Sotiriadis-Gambassi-Silva (2013)
  - CFT (only Loschmidt) Cardy (2014)
  - spin chains (only Loschmidt) Venuti et al. (2011), Fagotti (2013) Pozsgay (2014), De Luca (2014), Andraschko-Sirker (2014)
- Local quenches
- What about interactions? In 1D it can be very important.

Outline

- Preliminaries
  - Work statistics and Loschmidt echo
  - Partition function, TBA, bTBA
- Analytics
  - multiparticle expansion
  - low energy part
- Numerics
  - avoiding oscillatory integrals
- Conclusions

Sinh-Gordon

Relation to Loschmidt echo

e.g. in Talkner et al. Phys. Rev. E 75 (2007), 050102(R)

- Characteristic function of the work distribution

$$G(t) = \int dW e^{iWt} P(W) = \int dW e^{iWt} \sum_{nm} \delta(W - \bar{E}_m + E_n) \underbrace{p(m|n)}_{|\langle m|U|n\rangle|^2} p_n$$

$$= \sum_{nm} \langle m|U e^{-iuH_0}|n\rangle\langle n|U^\dagger e^{iuH_1}|m\rangle p_n = \mathrm{Tr} e^{-iuH_0} e^{iuH_{1,H}} \rho_0 = \langle T e^{iu\int_0^t \frac{\partial H_H(s)}{\partial s} ds}\rangle$$

Classical result: $\langle e^{iu\int_0^t \dot{\lambda}_s \frac{\partial \mathcal{H}(\lambda_s)}{\partial \lambda_s} ds}\rangle_{\mathrm{cl.}}$

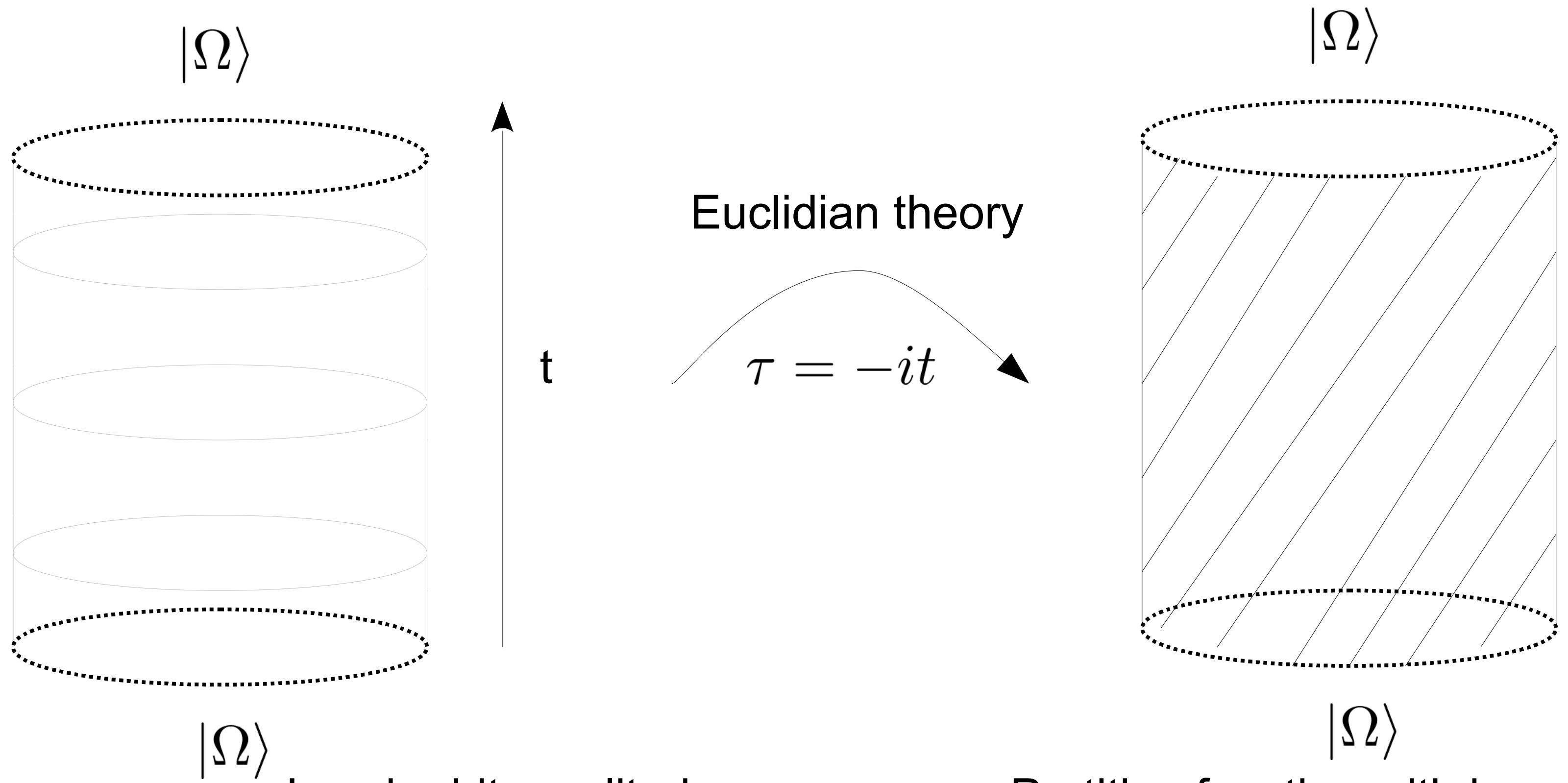
$$\to \langle \Omega|e^{-iuH_0} e^{iuH_1}|\Omega\rangle = L(u)$$

sudden quench limit, T = 0

- Provides a way to measure P(W) Dorner et al. PRL 110 (2013) 230601

Loschmidt amplitude

$$L(t) = \langle \Omega | e^{iHt} | \Omega \rangle \qquad\qquad Z(\tau) = \langle \Omega | e^{-H\tau} | \Omega \rangle$$



Loschmidt amplitude

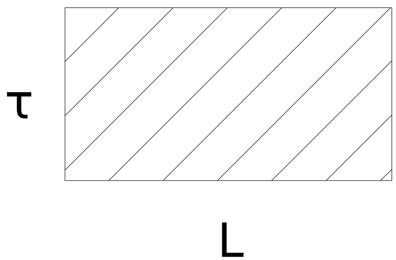
Partition function with boundaries

Idea: try to find Z and continue it: $L(t) = Z(-it)$

E.g. used for dynamical phase transitions by Heyl et al. (2013)

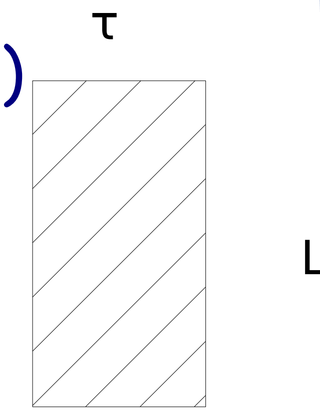
Partition function in 1d

- Finite volume L, finite temperature 1/τ (Eucl. time)



$$Z_L(\tau) = \mathrm{Tr} e^{-\tau H_L} = \mathrm{Tr} e^{-L H_\tau} = Z_\tau(L)$$

two equivalent quantization schemes



TDL in L: $$e^{-Lf(\tau)} \approx \sum_i e^{-\tau E_i(L)} = \sum_j e^{-L\bar{E}_j(\tau)} \approx e^{-L\bar{E}_0(\tau)}$$

free energy density
at finite temperature τ in TDL

Casimir energy
in finite volume τ

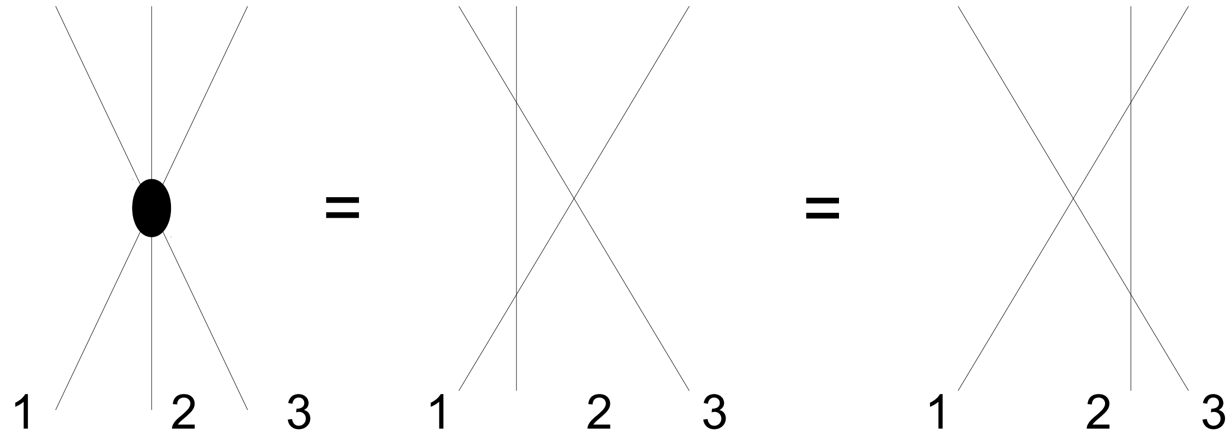
- With boundary: $Z_L(\tau) = \langle \Omega | e^{-\tau H_L} | \Omega \rangle \approx e^{-L\bar{E}_0^{\Omega\Omega}(\tau)}, \quad L \to \infty$
- Split $f(\tau)$: $f(\tau) = f_b \tau + 2 f_s + f_C(\tau)$

shift norm. shape

$$P(W) = \frac{1}{2\pi} \int dt e^{-iWt} e^{-Lf(-it)}$$

Integrability

- Factorized scattering



$$S(p_2,p_3)S(p_3,p_1)S(p_1,p_2) = S(p_1,p_2)S(p_1,p_3)S(p_2,p_3)$$

- 1+1 dimensions
- Infinite number of conserved charges
- Particle picture
- Elasticity
- Factorization

- Faddeev-Zamolodchikov operators

asymptotic states: $|\theta_1 \ldots \theta_n\rangle = Z^\dagger(\theta_1)\ldots Z^\dagger(\theta_n)|0\rangle$

$E = m\cosh\theta$

$p = m\sinh\theta$

$E^2 - p^2 = m^2$

$$Z(\theta_1)Z_(\theta_2) = S(\theta_1-\theta_2)Z(\theta_2)Z_(\theta_1)$$

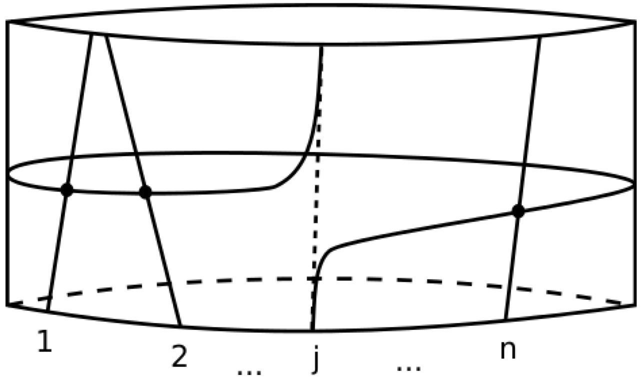
$$Z(\theta_1)Z_(\theta_2)^\dagger = S(\theta_2-\theta_1)Z(\theta_2)^\dagger Z_(\theta_1) + \delta(\theta_1-\theta_2)$$

(free bosons: S=1.)

Thermodynamic Bethe Ansatz

- Bethe-Yang equation (constraint):

$$e^{ip_i L} \prod_{j \neq i} S(\theta_i - \theta_j) = \pm 1, \quad L \to \infty$$



$$m \cosh\theta - i \int d\theta' \rho_r(\theta') \log S(\theta - \theta') = 2\pi \rho(\theta)$$

density of occupied states level density

- Partition function:

$$Z = \int [d\rho_r] \exp\left\{ -L \int m\tau \cosh(\theta) \rho_r(\theta) d\theta + S([\rho_r]) \right\} \approx e^{-L f_C(\tau)}$$

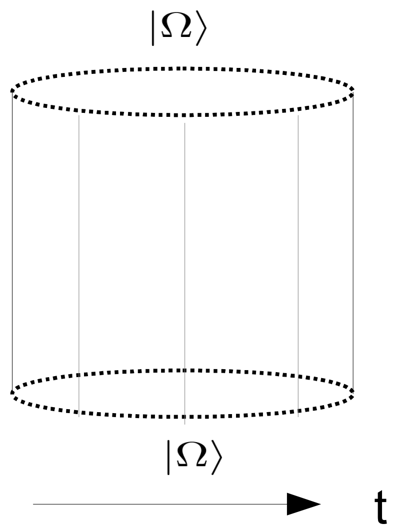
with $f_C(\tau) = -\frac{m}{2\pi} \int \cosh(\theta) H(\theta), \quad H(\theta) = \log(1 + e^{-\varepsilon(\theta)}), \quad e^{-\varepsilon} = \frac{\rho_r}{\rho_h}$

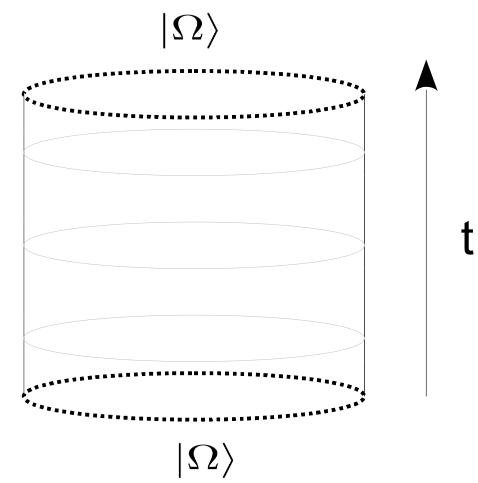
$$\varepsilon(\theta) = \tau m \cosh\theta + \Phi * H(\theta) \qquad f * g = \int f(x_1 - x_2) g(x_2) dx_2$$

$$y(\theta) = \Phi * H(\theta)$$

a nonlinear integral equation

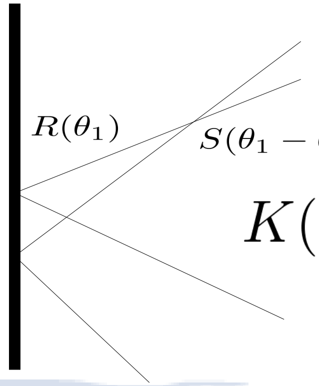
Boundary TBA





- Integrability preserving boundaries:

$$|\Omega\rangle \sim e^{\int K(\theta) Z^\dagger(-\theta) Z^\dagger(\theta) d\theta} |0\rangle$$



$$K(\theta) = R(i\pi/2 - \theta)$$

- Such initial states can be incorporated into TBA as a rapidity dependent chemical potential

LeClair et al. Nucl. Phys. B 453 (1995), 581

$$f_C(\tau) = -\frac{m}{4\pi} \int d\theta \cosh(\theta) H(\theta)$$

$$H(\theta) = \log(1 + |K(\theta)|^2 e^{-2\tau m \cosh\theta - y(\theta)})$$

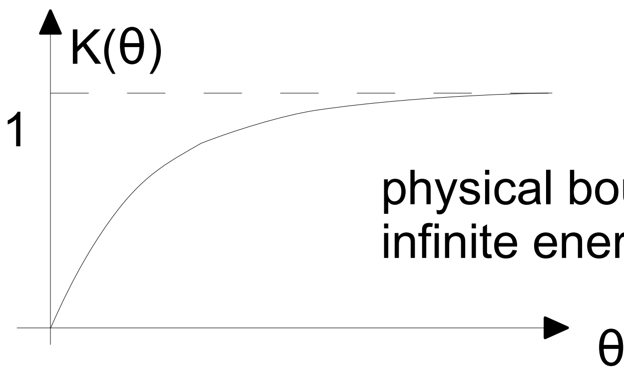
Initial states

$$f_C(\tau) = -\frac{m}{4\pi}\int \cosh(\theta)\log(1+|K(\theta)|^2 e^{-2\tau m\cosh\theta - y(\theta)})$$

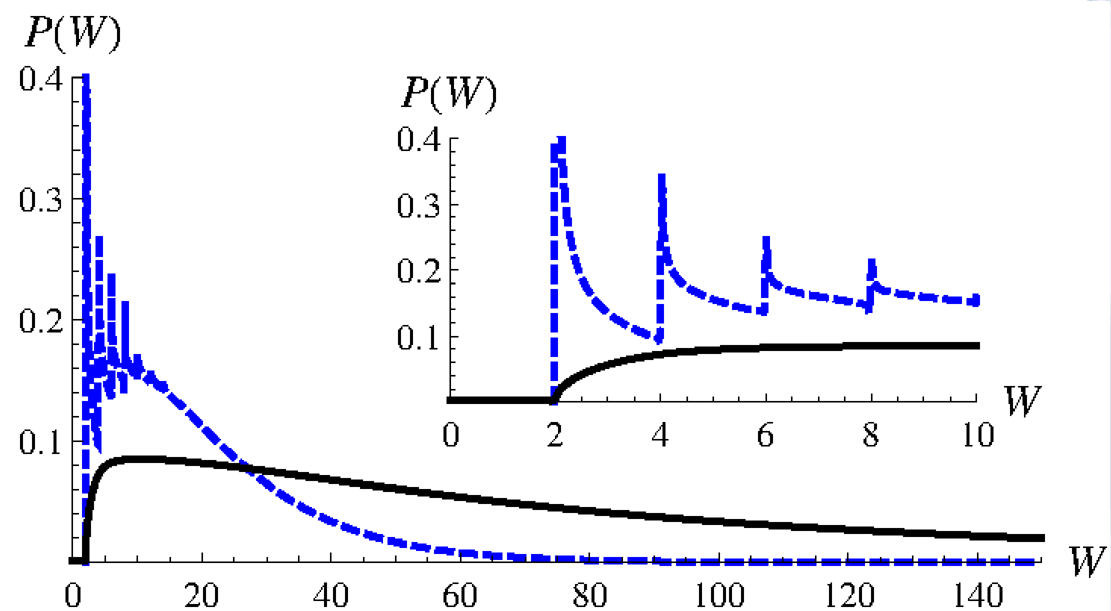
?

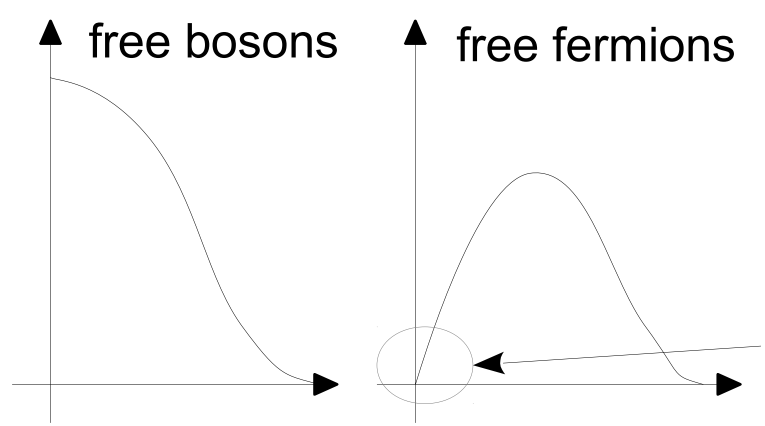
Depends on $Z_0(p)|\Omega\rangle = 0$ for free theories: $|\Omega\rangle \sim e^{\int K(\theta)Z^\dagger(-\theta)Z^\dagger(\theta)d\theta}|0\rangle$

Sotiriadis et al. Phys. Rev. E 87, 052129 (2013)



physical boundary
infinite energy – wall





similarity is
discouraged, Pauli principle

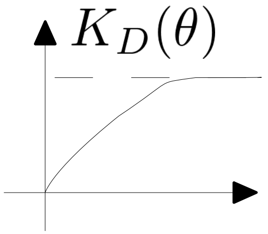
Initial states relative to QQs

Sotiriadis et al. Phys. Lett. B 734 (2014), 52

- Quenching from infinite mass:

  no field fluctuations, Dirichlet state:

  $$|\Omega\rangle \sim e^{\int K_D(\theta) Z^\dagger(-\theta) Z^\dagger(\theta) d\theta} |0\rangle$$



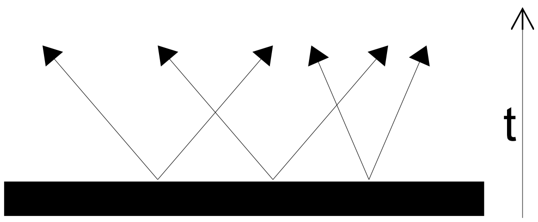
needs regularization

- In general: $Z_0(p)|\Omega\rangle = 0$

  for zero prequench interaction: $$\left(E_{0p}\phi(p) + [\phi(p), H]\right)|\Omega\rangle = 0$$

  - multiparticle expansion
  - infinte system of integral equations involving form factors
  - $m_0 \to \infty: \;\; K \to K_D K_{\text{free}}$

    $K(0) = 0$ like fermions



Results

Analytics: multiparticle expansion

bTBA

$$y(\theta) = \Phi * H(\theta)$$

$$y(\theta, it) = \int d\theta' \Phi(\theta - \theta') \log(1 + |K|^2 e^{-2mit\cosh\theta' - y(\theta', it)})$$

$$= \sum_{k=1}^{\infty} \sum_{\ell=0}^{\infty} \frac{(-k)^\ell}{k \ell!} \int d\theta' \Phi(\theta - \theta') |K(\theta')|^{2k} e^{-2kmit\cosh\theta'} y(\theta', it)^\ell = \sum_{n=1}^{\infty} y_n(\theta, it)$$

$$\mathcal{F}[y_n(\theta, it)](\theta, \omega) \sim \Theta(\omega - 2nm)$$

- multiparticle expansion

Iterative solution

$$y_{(0)} \equiv 0$$
$$y_{(n)} \equiv \Phi * H[y_{(n-1)}]$$

$$\begin{tikzcd}
y_{(1),1} \arrow[dr] \arrow[drr] & y_{(1),2} \arrow[dr] & y_{(1),3} \\
y_{(2),1} \arrow[dr] \arrow[drr] & y_{(2),2} \arrow[dr] & y_{(2),3} \\
y_{(3),1} & y_{(3),2} & y_{(3),3}
\end{tikzcd}$$

$$y_1 = y_{(1),1} = y_{(2),1} = \dots$$
$$y_2 = y_{(2),2}$$
$$y_3 = y_{(3),3}$$

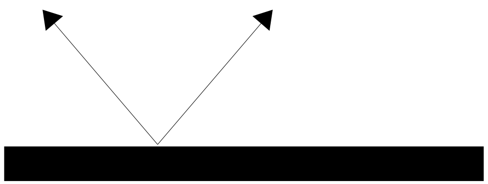
Lower terms become exact after a few iterations

Similar multiparticle expansions for $P(W),\ f_C(W)$

$$P(W) = e^{-2f_s} \left[\delta(W) + \sum_{n=1}^{\infty} p_n(W) \Theta(W - 2nm) \right]$$

Low energy part of P(W)

$$P(W) = e^{-2f_s}\left[\delta(W) + \sum_{n=1}^{\infty} p_n(W)\Theta(W-2nm)\right]$$



$$p_1(W) = \frac{mL}{4\pi}\int d\theta \cosh\theta |K(\theta)|^2 \delta(W - 2m\cosh\theta)$$

$$p_2(W) = \frac{m^2L^2}{32\pi^2}\int d\theta \int d\theta' \cosh\theta\cosh\theta' |K(\theta)|^2 |K(\theta')|^2$$
$$\times\left[1 + \frac{4\pi\delta(\theta-\theta')}{mL\cosh\theta'} - \frac{8\pi\Phi(\theta-\theta')}{mL\cosh\theta'}\right]\delta(W - 2m\cosh\theta - 2m\cosh\theta')$$

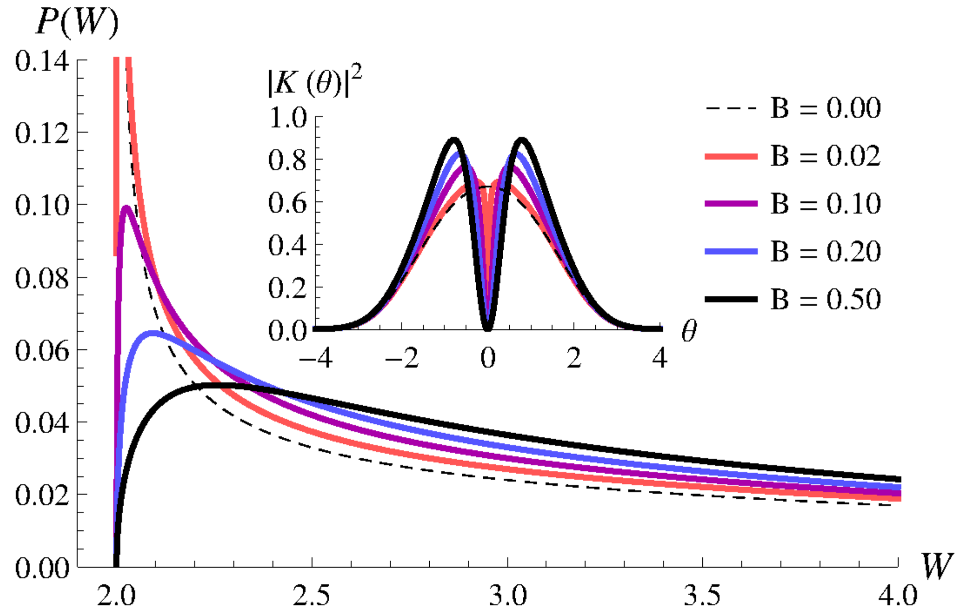
W < 4m: independent of Φ:
2 particles never scatter off each other in infinite volume

sinh-Gordon model

$$H = \frac{1}{2}\pi^2(x) + \frac{1}{2}\left(\frac{\partial\phi}{\partial x}\right)^2 + \frac{m^2}{b^2}(\cosh b\phi - 1)$$

$$S(\theta) = \frac{\sinh\theta - i\sin\frac{\pi B}{2}}{\sinh\theta + i\sin\frac{\pi B}{2}}, \quad B = \frac{b^2}{1+b^2}$$

- Genuine interaction
- Sine-Gordon with imaginary b
- One particle, simplest Toda



Numerics: oscillatory integrals

- W greater, exact solution becomes tedious. What about non-analyticities?
- A numerical approach is needed
- Complex τ, integrals highly oscillatory

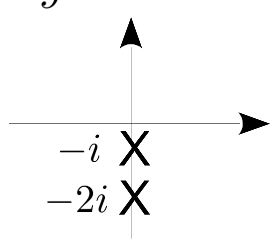
$$y(\theta,\tau) = \int d\theta' \, \Phi(\theta-\theta') \log\left(1+|K|^2 e^{-2m\tau\cosh\theta' - y(\theta',\tau)}\right)$$

$$f_C(\tau) = -\frac{m}{4\pi}\int d\theta \cosh(\theta) \underbrace{\log\left(1+|K|^2 e^{-2m\tau\cosh\theta - y(\theta,\tau)}\right)}_{H(\theta)}$$

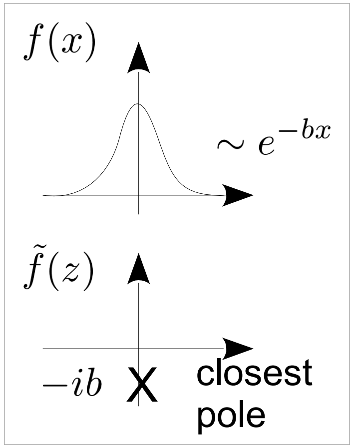
- Avoiding this integral:

$$\tilde{y} = \tilde{\Phi}\cdot\tilde{H}$$

$$\Phi(\theta) = \Phi_1 e^{-\theta} + \dots$$
$$H(\theta) = H_1 e^{-2\theta} + \dots$$



$$y(\theta) = \Phi_1 \underbrace{\tilde{H}(-i)}_{\int d\theta \cosh\theta H(\theta)} e^{-\theta} + \dots$$



- Alternative formula: $$f_C(\tau) = -\frac{m}{4\pi}\lim_{\theta\to\infty}\frac{y(\theta,\tau)}{\Phi(\theta)}$$

Numerics: sinh-Gordon model

- Trivial phase diagram, no branch cuts?
  No pole contributions picked up? Etc.
- Numerical approach: solve the original bTBA
- Free energy: continuous, Cauchy-Riemann satisfied
  - True analytic continuation

sinh-Gordon model

$$H = \frac{1}{2}\pi^2(x) + \frac{1}{2}\left(\frac{\partial\phi}{\partial x}\right)^2 + \frac{m^2}{b^2}(\cosh b\phi - 1)$$

$$S(\theta) = \frac{\sinh\theta - i\sin\frac{\pi B}{2}}{\sinh\theta + i\sin\frac{\pi B}{2}}, \qquad B = \frac{b^2}{1+b^2}$$

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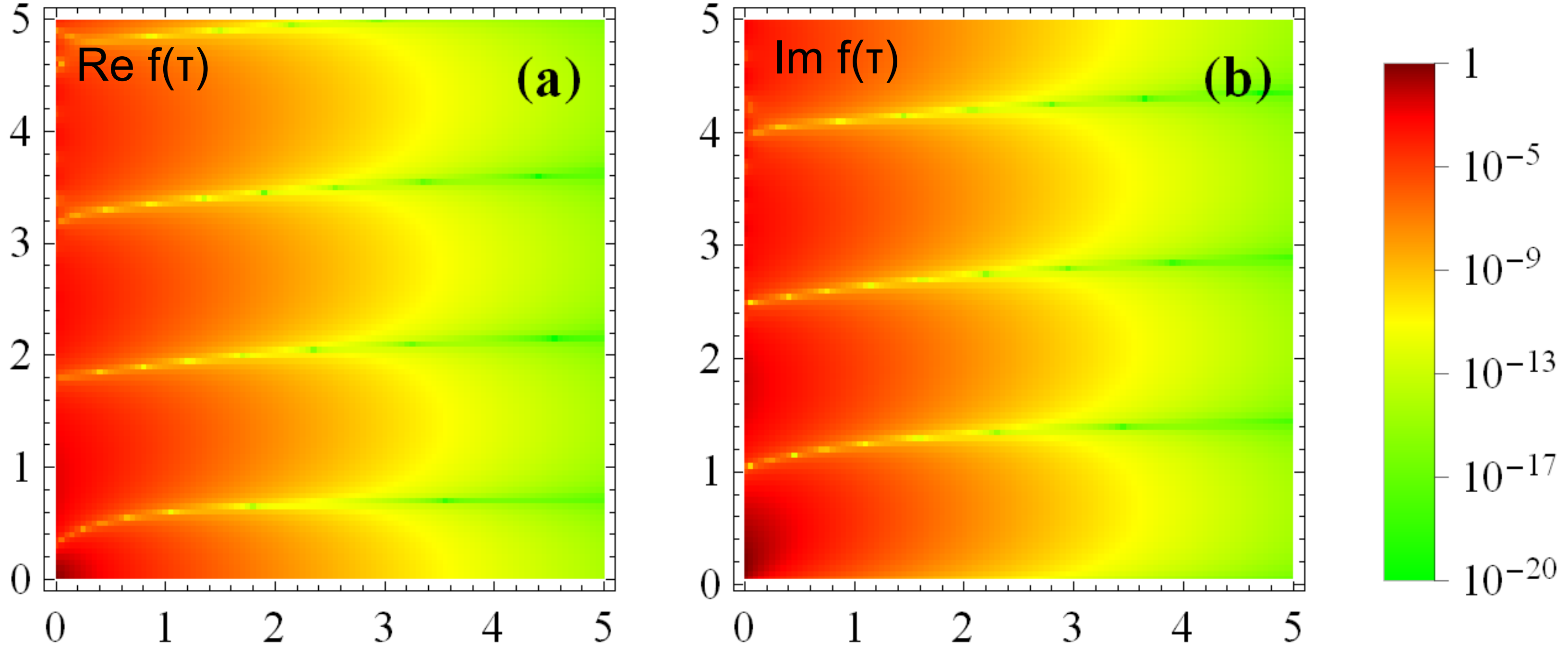
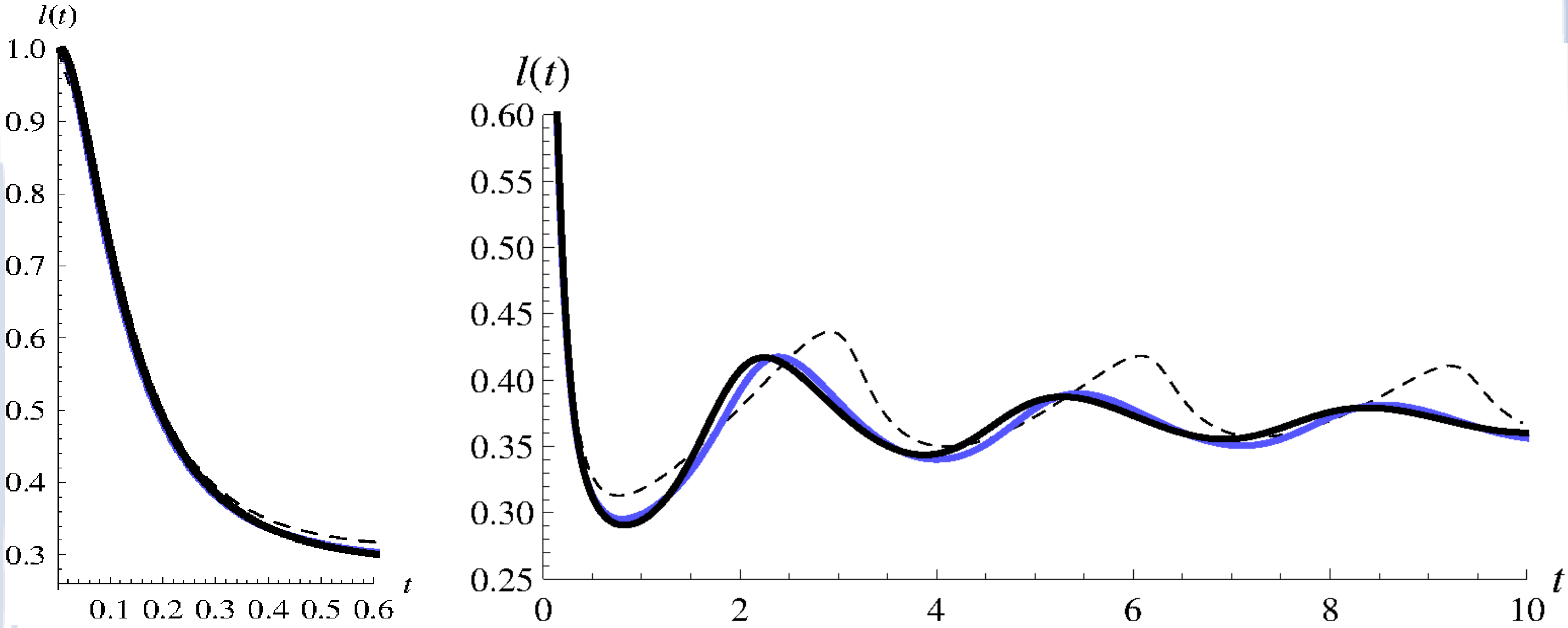


L = 5/m, m0 = 10m

Loschmidt echo

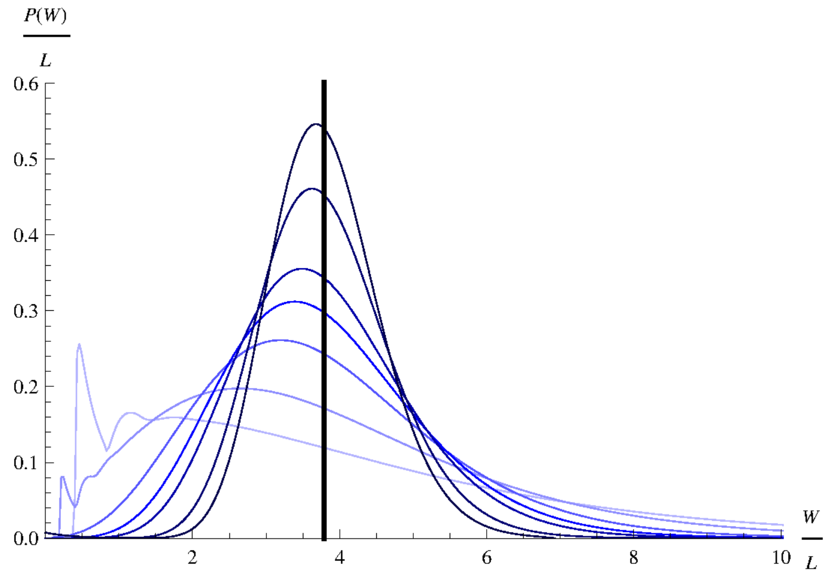
- Loschmidt echo per volume



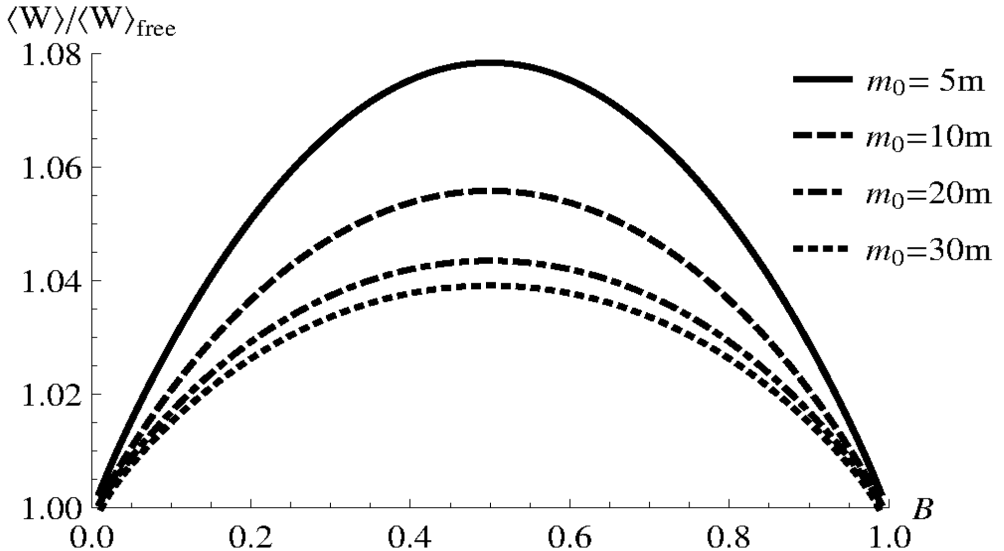
Work statistics - globally

- Extensivity of W

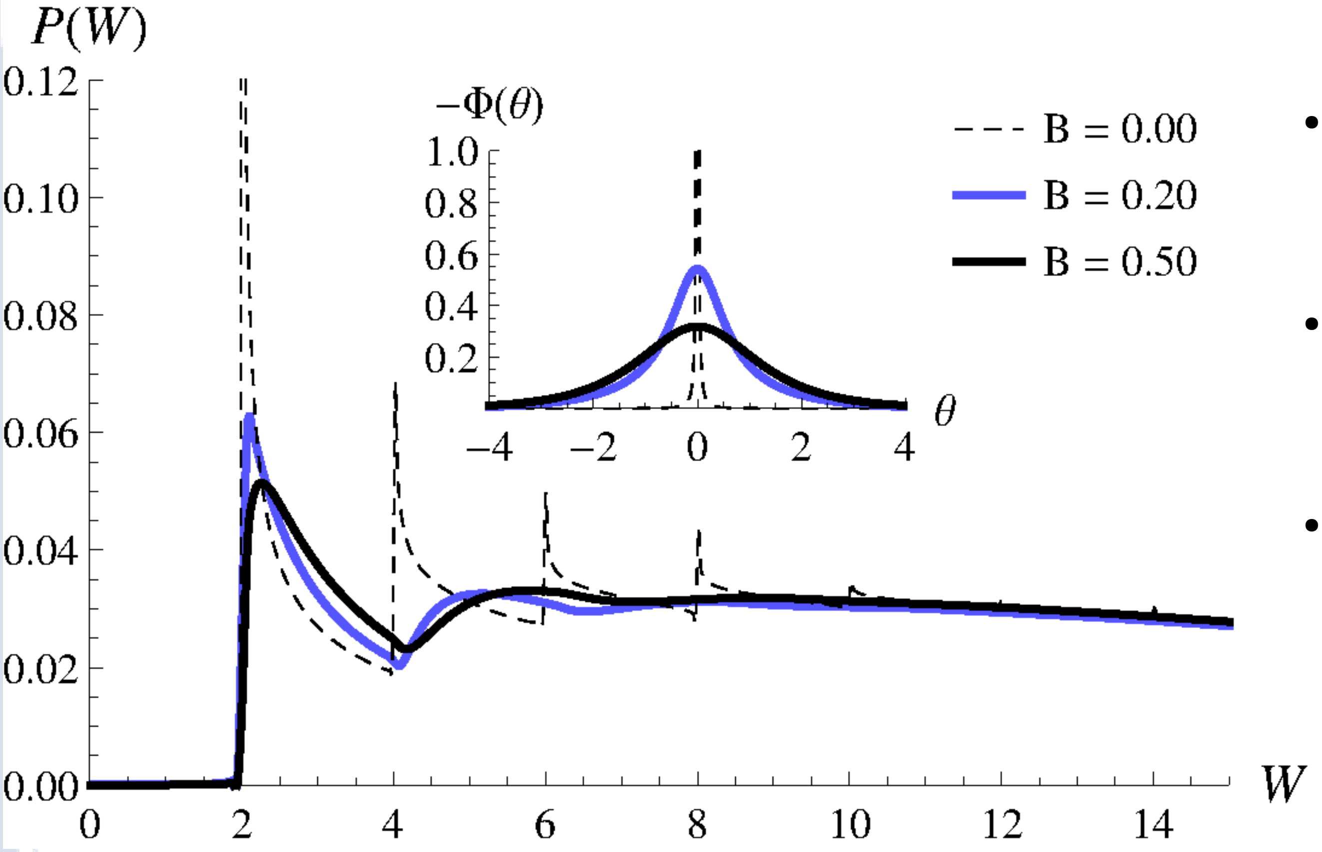
  L = 5, 10, 20, 30, 40, 70, 100



- Departure from free boson result



Work statistics - details



- Features of free fermion statistics appear
- Exponents change due to interaction, -1/2 to +1/2
- Not fermionic: there are multiple edges, however less pronounced

L = 5/m, m0 = 10m

Conclusions

- Quench performed on an integrable QFT, calculated Loschmidt echo, work statistics
- Nice application of boundary TBA – solving for imaginary temperature
- Low-energy part is given analytically through a multiparticle expansion and is model independent (for identical initial states)
- To perform numerical calculations proved a property of (b)TBA and used it to avoid oscillatory integrals for complex temperatures
- Investigated effects of turning on a genuine interaction term
  - free boson to sinh-Gordon
- Observed that the details of the statistics change considerably, developing fermionic features